Glass slippers and glass ceilings: An analysis of marital anticipation and female education.

Saqib Jafarey*
Department of Economics
City University
Northampton Square
London EC1V 0HB
United Kingdom

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ABSTRACT

This paper studies how the anticipation of marriage affects female education in the presence of gender wage inequality and private subjective benefits of education. The first induces a marital division of labor that creates (i) a marginal disincentive to girls’ schooling and (ii) a threshold level of domestic skills for females in marriage markets. The second leads to an externality which marginally favours both boys’ and girls’ schooling. We show that when the threshold effect dominates, economic growth can have negative consequences for female education and labor supply. This resembles an observed relationship between female employment and development.

Keywords: Female education, labor market discrimination, marriage.
JEL Classification: I20, J12, J16, O12.

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* E-mail address for correspondence: s.s.jafarey@city.ac.uk
1 Introduction.

It is broadly true that female children in developing countries receive less education than male ones and that differences in culture and levels of per-capita national income might modulate this bias but do not eliminate it.\(^1\) Indeed when tertiary education is factored in, it could be argued that such a bias exists even in the developed countries, remaining latent at lower levels of education.\(^2\)

There is also evidence of anti-female bias in child nutrition and healthcare (see, e.g. Khanna et. al. [2003]). A common economic explanation is that these biases represent optimal parental responses to gender inequalities in returns to labor and human capital (see Rosenzweig and Schultz [1984] for a seminal investigation of this hypothesis). Faced with lower returns to females, parents shift resources towards males.\(^3\)

At the same time, while it has been noted that gender biases in survival are mainly a matter of poverty and economic insecurity (see Deaton [1989] and Rose [1999]), those associated with education persist even when incomes rise above the poverty

\(^1\)See Dreze and Kingdon [2000], Grootaert [1998], Ilahi [1999], Ilahi and Sedlacek [2000], Ray [2000]. As an exception, Munshi and Rosenzweig [2004] find that among lower-caste Marathas, girls are more likely than boys to receive a modern English education, as opposed to a traditional Marathi one. The arguments of this paper are broadly consistent with both types of findings.

\(^2\)With tertiary education, the bias might appear in choice of subjects rather than years of study. Gender differences in fields of university study are widely observed, with women tending to specialise in subjects that are less rewarding in pecuniary terms than those typically chosen by men (see Folbre and Badgett [2003]).

\(^3\)In this paper, we shall assume that there is positive wage discrimination against females while noting that the empirical evidence for this hypothesis is mixed. For example, Kingdon [1998] and Nasir [2002] found evidence for lower returns to girls’ schooling in the case of India and Pakistan respectively, Behrman and Deolalikar [1995] and Aslam [2009] found the opposite to be true in the case of, respectively, Indonesia and Pakistan. Note however that our argument is not based on gender discrimination in returns to schooling alone but on how it induces and interacts with the household division of labour.
level. This could be because ensuring children’s survival is cheaper and involves fewer tradeoffs than providing them with education, so that while escaping extreme poverty might be sufficient to erase gender inequalities in child survival, it might not be enough to erase the anti-female bias in education.

One factor likely to influence educational choice is the expected use of time each child will make when he or she grows up. In particular the prospect that a child will marry on reaching maturity can be an important influence on the educational choices their parents make (on their behalf) when young. This paper analyses this relationship, arguing that marriage can create a distinct anti-female bias in education which exacerbates that arising from labor market inequalities alone.

The argument is made in three steps. In the first step, we show that the presence of gender wage discrimination induces a marital division of labor which encourages specialisation: a male is more likely to work in the market, a female more likely to work at home and even when one partner does not specialise, the other tends to.

In the second step, we show that the anticipated marital division of labor exerts a negative influence on female education. This influence is multiplicative in that a female who is expected to get married might be subject to an even stronger anti-education bias than her ‘single self’. The influence is discontinuous in that while a progressive reduction in gender wage inequality will progressively equalise outcomes for single males and females, it will not do so for their married counterparts.

In the final step, we analyse the marriage decision itself, under the assumption that young adults choose between getting married and staying single. We show that a member of the privileged gender will require a prospective partner to have a level of formal education which lies within an interval of acceptable values. This is because the privileged spouse expects to provide an income subsidy to his partner and expects sufficient household skills in return, ruling out females with education either above
or below the set of acceptable values. This ‘marital desirability’ constraint can limit female education even further. Indeed females with a high subjective preference for education might forsake marriage altogether, rather then restrict their education to meet this constraint.

A consequence of male-imposed constraints on female education is that, when female wages are low to begin with, a proportionate increase in male and female wages can lead to a decline in the education and labor supply of married females. This is because when female labor market participation is initially low, the implicit income subsidy from male to female will increase even if female wages have gone up in proportion to those of men. This will make the latter more demanding of their prospective partners’ skills at homemaking.

The argument that the prospect of marriage can create a downward pressure on female education and labor market participation is an anecdotal one. But to our knowledge it has rarely been subject to economic analysis. One paper to do so, Lahiri and Self [2007], argues that parents decide on their children’s education by taking account of their future contributions to household income. While boys are expected to remain part of the parental household after getting married, girls are expected to leave and join their in-laws’ household. Parents discount the future earnings of their daughters as they are expected to accrue to their in-laws and this leads to under-investment in daughters’ education.⁴

Lahiri and Self’s explanation is complementary to the one made in this paper, in which all children become financially independent of the parental household upon growing up so the attribution of future income is not of concern. The source of anti-female bias arises due to the anticipated division of labor for married couples.

⁴More recently, Lahiri and Self [2008] extend this line of argument to explain the anti-female bias in survival ratios. They argue that in the presence of costly health care, both labor market discrimination and an inter-household externality can lead to such a bias.
Folbre and Badgett [2003] have studied the effect of marriage on occupational choice, with the argument that the marriage market reinforces gender stereotypes by penalising both men and women who pursue non-traditional occupations. Testing this hypothesis by measuring student responses to personal advertisements, they found that both men and women who reported studying for or holding non-traditional occupations were rated as less attractive, holding other factors constant. Our analysis suggests an underlying reason for such preferences – a female in an atypical female occupation could suggest an educational background (or subjective preferences for certain types of education) which detracts from ‘desirable’ domestic skills, while men in atypical male occupations might signal insufficient market earning power.

More recently, Fisman et. al. [2006] have analysed the results of speed dating experiments in which participants are asked to accept or reject a potential partner for an extended date on the basis of a four-minute meeting. Relating the yes/no decisions of each participant to the attributes of their potential partners, they found that females were more likely to select male partners on the basis of higher intelligence, ambition while males were more likely to use physical attractiveness as a criterion. Moreover men did not value women’s intelligence or ambition when it appeared to exceed their own. This is consistent with our own result that a male might reject a female whose educational level exceeds a maximum threshold, normally less than his own.5

The rest of the paper is organised as follows. Section 2 describes the model. Section 5

5Neither Folbre and Badgett nor Fisman et. al. explain how matching preferences might arise. Folbre and Badgett assume that a partner’s prospective occupation is itself a criterion affecting attractiveness, while Fisman et. al. refer to work in psychology and sociology that has respectively emphasised the role of biology and tradition in creating gender roles and stereotypes that in turn influence sexual preferences. In our model matching preferences are based on the anticipated marital division of labor post-marriage: labor market discrimination is the sole factor underlying the male preference for females with sufficient homemaking skills and the female preference for males with income earning power.
3 analyses the labor market and educational decisions of single individuals. Section 4 analyses the analogous decisions for married individuals. Section 5 analyses the marital decision and section 6 offers concluding remarks.

2 The model.

There are two households, labelled $X$ and $Y$ respectively. The households have one offspring each, intrinsically identical save for their respective genders. Each offspring proceeds through two stages of life: childhood and adulthood. In each stage of life they have a time endowment equal to unity.

In the childhood stage, each parent decides on how to allocate the child’s time between formal schooling and domestic chores. This allocation determines the level of market versus household skills that the child grows up with.

Labeling the time spent in schooling as $s$, and the level of human capital as $e$, we assume that the latter is a linear function of the former: $e = \sigma s$. $\delta$ represents the maximum level of education that can be attained.

2.1 Homemaking versus market skills.

Time spent in domestic chores, $1 - s$ leads to the formation of adult household skills, labelled $\alpha$. We assume that the relationship between $\alpha$ and $1 - s$ is represented by a function $\alpha = \alpha(1 - s)$ such that:

\begin{align*}
(A-1) \quad & \alpha(0) \equiv \alpha_0 \geq 0, \quad \alpha'(0) > 0; \quad \alpha(1) \equiv \alpha_1 > 0, \quad \alpha'(1) < 0. \\
(A-2) \quad & \alpha''(1 - s) < 0; \quad \forall s \in [0, 1].
\end{align*}

\footnote{It is common in the literature on schooling and human capital to assume a linear relationship but the results do not depend on this assumption.}
(A-3) \exists \hat{s} \in (0, 1] such that \( \alpha(1 - \hat{s}) \equiv \bar{\alpha} = \max\{ \alpha(1 - s) \} \forall s \in [0, 1]. \)

Assumption (A-1) implies that even without any household training when young, each adult will possess some household skills, but that starting from such a corner, time spent in household training will increase these skills.

Assumption (A-2) states that this relationship is strictly concave; thus, assumption (A-3) guarantees that a child who combines domestic training with some formal schooling can grow up having more household skills than one with either no schooling or too much schooling. This is because formal schooling can instil basic skills, such as literacy and numeracy, which are useful for housework. The degree of complementarity between formal education and household skills, and thus the turning point in the formation of household skills can vary according to the social stratum to which the children belong. At higher social levels, not only literacy and numeracy but also exposure to the arts, history and literature might be important for the development of domestic skills. What is important is that there is some point at which further development of market skills comes into conflict with the formation of household skills.\(^7\)

At any value of \( s \), then, \( \alpha \) and \( e \) are uniquely related. This relationship is characterised by the function:

\[
\alpha(e) = \alpha \left( \frac{\sigma - e}{\sigma} \right)
\]

where \( \alpha = \alpha_0 \) when \( e = \sigma \), \( \alpha = \alpha_1 \) when \( e = 0 \) and \( \alpha = \bar{\alpha} \) when \( e = \hat{e} = \sigma \hat{s} \).

Figure 1 below shows the combinations of household skills and formal education that result from any given choice of \( s \).

\(^7\) (A-2), (A-3) and the assumption that \( \alpha'(0) > 0 \) are included for generality and are not necessary for the results of this paper. These require only that there is some interval of values of \( s \) over which homemaking skills decline with \( s \).
The horizontal axis measures time in schooling, $s$, which lies between zero and unity. The vertical axis on the left measures formal education, $e$, which lies between zero and $\sigma$, increasing linearly at the rate $\sigma$. The vertical axis on the right measures housekeeping skill, $\alpha$. Time spent in housework as a child decreases as $s$ rises. Accordingly, at $s = 1$, $1 - s = 0$ and $\alpha = \alpha_0$. At $s = 0$, $\alpha = \alpha_1$. For intermediate values of $s$, $\alpha$ first increases and then decreases after peaking at $\bar{\alpha}$.

In the second period, the adult decides whether to marry or not, moves out accordingly and makes a time-allocation decision between market work ($\ell$) and housework $(1 - \ell)$. This decision is made conditional on the individual’s marital status and hourly market wage. The latter in turn depends on an underlying wage $\omega_i$ and the individual’s human capital $e_i$ through the linear function:

$$w_i = \omega_i e_i$$  \hspace{1cm} (1)

where $w_i$ represents the hourly wage, and $\omega_i$ is an underlying wage per unit of human capital.

Wage inequality exists if $\omega$ is not the same for both genders. In this paper we take this possibility to be exogenous.

### 2.2 Preferences and constraints.

We assume that childhood utility is separable from adult utility and does not depend on the choice between schooling and household chores. Each adult has a utility function:

$$U_i = u(c_i) + h_i + \delta z + be_i$$  \hspace{1cm} (2)
\( i = X, Y; u(\cdot) \) is a concave function of the adult’s own consumption of a market good, satisfying \( u'(0) = \infty; h_i \) is the utility from consumption of a household good; \( z \) is an index variable which takes on the value 0 if the agent remains single, and the value 1 if the agent gets married; \( \delta \geq 0 \) is the utility of being married.

The last term in the utility function represents a non-marketable subjective benefit that the adult derives from education.\(^8\) Such a subjective benefit does not exclude the marketable pecuniary benefit; the latter is already incorporated in equation (1). The point is that education confers subjective benefits on top of marketable ones.\(^9\) These benefits could be direct, \textit{i.e.} education might confer pride and satisfaction for its own sake, or indirect, \textit{i.e.} education might enable individuals to seek out information which leads to better consumer choices. One example of the latter effect is the link between education and health, itself a source of subjective utility.\(^{10}\)

Each adult possesses one unit of time which can be split between market work \( \ell \) and housework \( 1 - \ell \). The level of market consumption depends on the individual’s market income. For a single individual, this is:

\[
c_i = w_i \ell_i
\]

where \( \ell \) denotes the fraction of adult time in market work.

\(^8\)Indeed we could equally assume that each spouse’s education confers some subjective benefits to the other by linking \( \delta \) to each spouse’s education, i.e. let \( \delta_i = \delta e_j^m \), where \( \delta_i \) is the happiness felt by spouse \( i \), and \( e_j \) is the education level of spouse \( j \). The main results of this paper, including those of section 5, would go through with slight modification, so long as (i) each parent \( j \) disregards the effect of \( e_j^m \) on \( \delta_i \), and (ii) \( b > \delta \), i.e. the subjective own-benefit exceeds the subjective cross-benefit.

\(^9\)See, \textit{e.g.} Schaafsma [1976] and Lazear [1977] for early attempts to disentangle the two effects of education.

\(^{10}\)As summarised in Cutler and Lleras-Muney [2006], a large body of research shows a robust positive relationship between health and education, even after the market returns of education have been controlled for. Their argument is that education leads to better decision-making and information-seeking and thus helps individuals maintain good health.
If married, the market good is assumed to be split equally between the two partners:\(^\text{11}\)

\[
c_X = c_Y = \frac{w_X \ell_X + w_Y \ell_Y}{2}
\]  

(4)

The utility from consumption of a household good depends non-linearly on the amount of effort put into household production according to a concave function. For a single individual, the utility is given by

\[
h_i = h(\alpha_i (1 - \ell_i))
\]  

(5)

where \(\alpha_i\) is the individual’s level of household skills and \((1 - \ell_i)\) is adult time spent in household production. We assume that \(h' > 0\) \(h'' < 0\) and that \(h'(0) = \infty\).\(^\text{12}\)

For married adults, we assume that there is (i) a single household production function per couple; (ii) their efficiency-adjusted effort levels are perfect substitutes for each other and (iii) the utility from the household good is equally shared, even when the good itself is rivalrous.\(^\text{13}\)

\[
h_X = h_Y = \gamma h(\alpha_X (1 - \ell_X) + \alpha_Y (1 - \ell_Y))
\]  

(6)

Here \(\gamma\) measures the degree of rivalrousness of the household good. \(\gamma = 1\) implies that the household good is completely non-rivalrous, while \(\gamma = 0.5\) implies it is completely rivalrous. Although much of the literature on household economics assumes that household goods and services are non-rivalrous, any value of \(\gamma\) strictly greater than 0.5 will meet this assumption, so we treat \(\gamma\) as an open parameter.\(^\text{14}\)

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\(^{11}\)This would result anyway from unitary decision-making and identical preferences, as is the case here.

\(^{12}\)Note that function \(h(\cdot)\) can be interpreted two ways: either that the household production technology is itself concave or that it is linear but the utility of household goods is concave.

\(^{13}\)The last assumption is again the result of unitary decision making under symmetric preferences.

\(^{14}\)Intuitively, some aspects of household production seem more rivalrous than others: for example, ironing or making sandwiches should be more rivalrous than gardening or house-cleaning.
2.3 Decision making.

We first impose the state of being married or remaining single on the adults and within each state we solve backwards for their time allocation decisions. In solving for adult time allocation we take as given the level of childhood schooling; then in solving for the latter we take into account its effect on adult time allocation.

Once the time allocation decisions have been solved for each state of marital status we consider the marital decision itself. We consider a non-consensual situation in which marriage is imposed exogenously on the two individuals, with a mechanism in which either party can veto the match.

Under the latter mechanism, we solve the second-stage problem of the marital decision by evaluating the utility from getting married or remaining single for each adult, given optimal time allocation decisions in each marital state. For marriage to take place, neither party should be worse off getting married than remaining single. Otherwise, one or both of them will veto the match.

3 Time allocation when single.

On reaching adulthood, the agent’s human capital $e$ is fixed as are $\alpha(e)$, $be$ and $w = \omega e$ (agent subscripts are suppressed since the problem is qualitatively identical for both). The adult maximises utility with respect to labor market participation, $\ell$.

Plugging the adult budget constraints into the utility function, the maximisation problem is expressed as :

$$\max_{\ell} = u(\omega \ell) + h(\alpha(e)(1 - \ell)) + be$$
which has first-order condition:

\[ u'(\cdot) \omega e - \alpha(e) h'(\cdot) = 0 \]  

(7)

For the single agent, specialisation between housework and market work is not an option because of the Inada conditions assumed above. Hence, the first-order condition holds as an equality.

The interpretation is analogous to the one in the standard case of endogenous labor supply when leisure counts for its own sake. Here, we ignore that aspect of leisure and the trade-off becomes one between market and home labor. A small increase in market labor increases utility from the market good by \( u'(\cdot) \omega e \) but reduces that from the home good by \( h'(\cdot) \alpha \). At the optimum, the two effects cancel out.

The resulting solution can be expressed as \( \ell(\omega, e) \). As is well known, a non-monotonic relationship between labor supply and the market wage is possible. A necessary and sufficient condition to rule this out is:

\[ (A-4) \quad u'(c) + u''(c) \cdot c > 0 \]

Under (A-4) it can be established that \( \ell_{\omega} > 0 \) (see Lemma 3, Appendix). Since changes in \( e \) can also affect equation (7) through the home production function, (A-4) is by itself not sufficient to rule out \( \ell_e \leq 0 \) but imposing a further sufficient condition ensures that \( \ell_e > 0 \):

\[ (A-5) \quad h'(\alpha(1 - \ell)) + h''(\alpha(1 - \ell))\alpha(1 - \ell) > 0 \]

(A-4) and (A-5) are in line with conventional restrictions imposed to prevent ‘backward bending’ labor supply and allow us to set benchmarks for comparing time allocation across genders and marital states.

The decision on education is taken by parents on behalf of the agent, taking into account the implications of childhood education on adult time use. The problem can
be expressed as:

$$\max_{e} = u(\omega \ell(e)) + h(\alpha(e)(1 - \ell(e))) + be$$

The first-order condition is:

$$u'(\cdot)\omega \ell + b + h'(\cdot)(1 - \ell)\alpha'(e) \geq 0 \quad (8)$$

The first-order condition can only be satisfied at $e \geq \hat{e}$. But there is no incentive to choose $e < \hat{e}$ since $\hat{e}$ is the amount of education where household skill $\alpha$ is maximised. If the first-order condition is satisfied with equality, $e \leq \sigma$. If, as a strict inequality, $e = \sigma$. The intuition is that a small increase in education will increase the utility from consumption, at given wages and market labor supply by an amount $u'(\cdot)\omega \ell$ and the subjective utility from education by $b$, while the utility from home production will fall, at given levels of home work and production, due to a fall in home skills $\alpha$ by an amount $\alpha'(e)$. Again these marginal effects cancel out at an interior optimum.

Once again, (A-4) is necessary and sufficient for $\partial e/\partial \omega > 0$ (see Lemma 3, Appendix). It is also easy to show that $\partial e/\partial b > 0$.

It is established in the Appendix, Lemmas 1-3, that (i) the relationship between $\ell$ and $e$ is upward sloping along both first-order conditions; (ii) under concavity of the maximisation problem, the $\ell - e$ locus is steeper along the first-order condition for education than under that for labor and (iii) an increase in the underlying wage causes the $\ell - e$ locus for optimal labor to shift upwards and that for optimal education to shift rightwards.

These results are depicted in Figure 2 below:

[Figure 2 in here]
The solid lines represent the first-order conditions, FOC$_e(w)$ and FOC$_\ell(w)$ respectively, at the lower wage, $w$, and the broken lines represent the first-order conditions at a higher wage, $w'$. Since the shifts in the two curves reinforce each other it is unambiguous that an increase in the underlying wage will cause both labor supply and education to increase.

We can now state the benchmark result:

**Proposition 1:** Under (A-4) and (A-5), if $\omega_Y \leq \omega_X$, and agents remain single, then $Y$ will have less education and do less market work than $X$, i.e. $e_Y \leq e_X$ and $\ell_Y \leq \ell_X$.

In other words, wage discrimination leads to asymmetries in education and time allocation across single people. At the same time, the degree of asymmetry is continuous in the degree of wage discrimination. As wage discrimination gets smaller so does the asymmetry in outcomes.

We shall refer to the “single self” of a married agent as the same agent had he or she not got married and likewise, the “married self” of a single agent as that agent had he or she got married. In referring to decisions made during childhood in anticipation of the future marital state, we shall refer to them as the “prospectively single” and “prospectively married” selves.

Optimal choices of a single self are denoted by $e^{s*}_i$ and $\ell^{s*}_i$, $i = (X,Y)$. Optimal choices of a married agent will be denoted as $e^{m*}_i$ and $\ell^{m*}_i$, $i = (X,Y)$. 

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4 Time allocation when married.

If married, the adults jointly decide on their respective levels of labor market participation.\(^{15}\)

The household utility function is expressed as:

\[
V = u(c_X) + u(c_Y) + h_X + h_Y + b \cdot e_X + b \cdot e_Y + 2\delta
\]

Since education levels are given at this stage, only the first four terms of the joint utility function are affected by the choice of labor market participation. As argued before, with unitary decision-making both partners will consume equal amounts of both the market and the household good so the problem reduces to:

\[
\max_{\{\ell_X, \ell_Y\}} V = 2u \left( \frac{\omega_X e_X \ell_X + \omega_Y e_Y \ell_Y}{2} \right) + 2\gamma h \left( \alpha(e_X)(1 - \ell_X) + \alpha(e_Y)(1 - \ell_Y) \right)
\]

which has first-order conditions:

\[
u'() \omega_i e_i - 2\gamma \alpha(e_i) h'() \geq 0 \quad i = (X, Y) \tag{9}\]

In choosing education, we assume that the parents care only for the welfare of their own offspring. We also assume that parents take into account the effect of their own child’s education on own adult labor supply but not that on the spouse’s. The latter is not essential since the envelope condition would apply even to the cross effects from education to adult time-use. Each parent then maximises the following analogous

\(^{15}\)We assume unitary decision-making within the marital household. The alternative non-unitary approach assumes that each member of the household makes strategic choices in their own self-interest. Basu [2006] has shown that with non-unitary decision-making, the allocation of household resources depends crucially on each members contribution to household income. Rainer [2006] has shown that a non-unitary mechanism is if anything even more likely to induce asymmetries within the household when combined with gender wage discrimination.
max \[u \left( \frac{\omega_i e_i \ell_i(e_i) + \omega_j e_j \ell_j}{2} \right) + \gamma h(\alpha(e_i)(1 - \ell_i(e_i)) + \alpha(e_j)(1 - \ell_j)) + be_i; \quad i, j = (X, Y)\]

The first-order condition is (terms involving the effect of \(e\) on adult labor supply drop out when evaluated at the optimum):

\[u'(\cdot)\omega_i \ell_i + 2\gamma h'(\cdot)(1 - \ell_i)\alpha'(e_i) + 2b \geq 0 \quad i = (X, Y) \quad (10)\]

Suppose initially that there is gender wage equality so that \(\omega_X = \omega_Y\), and assume for a moment that this induces an identical level of education for both children. Then the first-order conditions with respect to adult time can be combined for the two adults.

\[\frac{\omega_X e_X}{\alpha_X} = \frac{\omega_Y e_Y}{\alpha_Y} \geq \frac{2\gamma h'(\cdot)}{u'(\cdot)}\]

The weak inequality on the extreme right must hold as an equality for at least one of the two adults; but if it does hold as an equality, it must do so for both spouses implying that they divide their time identically. Hence, \(\ell_X = \ell_Y\).

Given that \(\ell_X = \ell_Y \equiv \ell\), the first-order conditions for education will also be identical and \(e_X = e_Y \equiv e\). Hence the married couple will be identical in both respects. In a companion paper (Jafarey [2008]) we analyse this case more fully and compare the outcomes for married adults with their single selves in the absence of gender wage inequality.

Now suppose \(\omega_X > \omega_Y\) and call agent \(Y\) the underprivileged spouse (or prospective spouse, when describing childhood decisions) and \(X\) the privileged spouse. Let us for now assume that when \(\omega_X > \omega_Y\), it follows that \(\omega_X e_X/\alpha(e_X) \geq \omega_Y e_Y/\alpha(e_Y)\). This will be established later, for now accept it to be the case.

Given the Inada conditions on utility functions, the following possibilities exist:

\[i) \quad \frac{\omega_X e_X}{\alpha_X} > \frac{\omega_Y e_Y}{\alpha_Y} = \frac{2\gamma h'(\cdot)}{u'(\cdot)};\]
In case (i) \( \ell_X = 1 \) while \( \ell_Y \geq 0 \); in case (ii) \( \ell_X = 1 \) while \( \ell_Y = 0 \); in case (iii) \( \ell_X \leq 1 \) while \( \ell_Y = 0 \).

It is immediately obvious that the underprivileged spouse will do less market work than the privileged spouse. The latter either specialises in market work or when he does not, the former specialises in house work.

To establish that 
\[
\frac{\omega_X e_X}{\alpha_X} \geq \frac{\omega_Y e_Y}{\alpha_Y},
\]

it is sufficient to show that \( e_X \geq e_Y \). In the first two cases, with \( \ell_X = 1 \), \( e_X = \sigma \), so the inequality follows trivially. In the third case, with \( \ell_X < 1 \), note that \( \ell_Y = 0 \) so the optimal value of education for agent \( Y \) satisfies
\[
2\gamma h'(\cdot)(1 - \ell_Y)\alpha'(e_Y) + 2b = 0
\]

Plugging in the value of \( e_Y \) that satisfies the above into the first-order condition for agent \( X \)’s education
\[
u'(\cdot)\omega_X \ell_X + 2\gamma h'(\cdot)(1 - \ell_X)\alpha'(e_Y) + 2b > 0
\]

Since \( \omega_X > \omega_Y \), \( \ell_X > \ell_Y = 0 \), \( (1 - \ell_X) < (1 - \ell_Y) = 1 \) and the other terms are identical for both spouses, it is clear the above equation is strictly positive when evaluated at \( e_Y \). Thus \( e_X > e_Y \).

How do the labor market effort and educational levels of the married female compare with that of her single self\footnote{The comparison between the married male and his single self is analysed in (Jafarey [2008]) where it is shown that so long as the male wage is sufficiently high or the public good nature of housework is not too large, then the married male will do more market work and and receive more education than his single self in all possible cases.}? In the two cases where she does no market work, it is obvious that her labor supply will be less than that of her single self since the latter
self undertakes positive levels of market work no matter how low the wage. In the case where both the married and single selves do market work, it can be shown that taking the education level to be given, the married self of agent $Y$ will do less market work than her single self. This is derived in the Appendix (Lemma 4).

The comparison is less clear-cut for education itself, especially when $b$ is large, because of a pro-education effect that marriage creates in that case. Intuitively, it arises because of an externality involving education that each prospective spouse brings to marriage.

Imagine that starting from some initial level of $e$, a parent marginally increases her child’s education, holding constant the child’s anticipated market participation. There will be three effect on welfare: (i) a positive effect from higher future earnings (weighted by $\omega\ell$); (ii) a negative effect from lower household skills (assuming that initially $e$ was on the downward sloping part of the skills tradeoff); (iii) a positive effect from greater subjective happiness (weighted by $b$). When a child is to remain single, all three effects will be internalised by the parent in reaching an optimal decision.

When a child is to be married, however, the first two effects will only be partially internalised as they will be shared with the prospective spouse.\(^{17}\) Only the third is fully internalised and the greater the value of $b$ the greater its weight in reaching an optimal decision. Of course, this does not mean that a married woman will always be more educated than her single self since their levels of market work will be markedly different, and this difference will tend to slant the comparison the other way. In other words, when $b$ is large, there can be some ambiguity in the comparison of education levels between the married and single selves of the same female.\(^{18}\)

If we therefore restrict attention to the case in which $b = 0$, then it can be established

\(^{17}\)If $\gamma = 1$, the effect of reduced household skills will be fully internalised by the parents but for all other values of $\gamma$, it will only partly be internalised.

\(^{18}\)This comparison is studied in more detail in Jafarey [2008].
that her education will also be less than that of her single self. First, in the case where she does no market work as an adult, her education will be at the peak of the market-household skills tradeoff, $\hat{e}$, trivially less than that of her single self. When she does some market work as an adult, it can be shown (see Lemma 5 in the Appendix) that, for given levels of market work, her education will be (i) less than that of her single self and (ii) this gap will be greater as the public benefit from home production gets greater.

Putting the results of Lemma 4 and 5, with those of Lemmas 1, 2 and 3, it can then be shown diagramatically that the married self of the the under-privileged spouse does less market work and receives less education than her single self, when both are chosen optimally. This is shown in Figure 3.

According to Lemma 4, the locus depicting the married self’s optimal labor (FOC$^m_{\ell_Y}$) lies always below that for her single self (FOC$^s_{\ell_Y}$) while according to Lemma 5, the analogous locus for the married self’s education (FOC$^m_{e_Y}$) lies always to the left of that for her single self (FOC$^s_{e_Y}$). Thus the optimal values, $\ell^*_Y < \ell^*_X$ and $e^*_Y < e^*_X$.

Although the above result has been derived by keeping $b = 0$, for the case of a high $b$, in the next section we show that the demands of a consensual marriage might restrict a married woman’s education in any case, even if her optimal choice as a married woman were to exceed that of her single self.

Finally note that the labor supply of married adults and the education of prospectively married children reacts discontinuously to gender wage discrimination. In case (i) above, both spouses work in the labor market, but $X$ specialises in this activity so that $\ell^*_X = 1, e^*_X = \sigma$ while $Y$ does not so that $\ell^*_Y < 1$ and $e^*_Y < \sigma$. Suppose now
that wage discrimination becomes progressively less, so that \( \omega_Y \rightarrow \omega_X \) from below. While \( \ell^m_Y \) and \( e^m_Y \) will rise, \( \ell^m_X \) and \( e^m_X \) will remain at unity.

Similarly in cases (ii) and (iii), \( \ell^m_Y = 0 \), \( e^m_Y = \hat{e} \), and a reduction of wage discrimination will not move these values away from the corner.

At the same time, if there is no wage discrimination to begin with, then \( \omega_Y = \omega_X \) and it can be shown that \( 1 > \ell^m_X = \ell^m_Y > 0 \) and \( \sigma > e^m_X = e^m_Y > \hat{e} \). This shows that due to the possibility of specialisation, even a small amount of wage discrimination can discontinuously induce corner solutions in the time use and education levels of one or both married agents. This is not true for their single selves, since their labor market and education levels vary continuously and identically with their respective wages. Thus as \( \omega_Y \rightarrow \omega_X \), the time allocation and education of single agents converge to each other.

The above results are summarised in the following.

**Proposition 2:** When \( \omega_X > \omega_Y \), (i) a married female works less than her single self and, if the subjective benefit of education is not too large, a prospectively married female receives less education than her prospectively single self; (ii) the time allocation and education of married agents are discontinuous in the degree of wage inequality while those of single agents are not.

5 The marital decision.

In this section we consider the marital decision. For marriage to take place, the following must be true:

\[
U^m_i \geq U^s_i \quad i = X, Y;
\]

\footnote{The details are in Jafarey [2008].}
i.e. neither agent should be made worse off by marrying than by remaining single. Rather than provide an exhaustive account of all the possible cases that might arise, we shall make some general observations about the factors that influence this decision and note some possibilities.

If wages were equal, then the two main factors making marriage attractive would be: (i) $\delta$, the companionship bonus and (ii) $\gamma$, the degree of non-rivalrousness of household production. A high value of either will increase the benefits of marriage.

Indeed, if $\delta = 0$ and $\gamma = 0.5$, both agents will (weakly) prefer to remain single. This is proven in the Appendix, Lemma 6. The intuition is that when $\gamma$ is low, the single self of each agent could, by mimicking the optimal time allocation of his or her married self, achieve exactly the same utility from consumption of the market good, and more from consumption of the household good. This is because the household production function itself is concave in time spent so that when married agents double up on housework they achieve less than twice their individual efforts. Unless what they produce is sufficiently non-rivalrous, they will be better off without cohabiting.

In the case of gender wage inequality, a third dimension arises in terms of the gains from marriage. This is because marriage between asymmetric adults creates comparative advantage in a way that is not possible with symmetric adults. Asymmetry between potential marriage partners can creep in through gender wage discrimination and then get reinforced by inequalities in education and time allocation. As a result of both the wage and the educational gap, the male partner has a comparative advantage in market work and the female in housework. This can make sharing a household mutually beneficial even absent companionship or any public good benefits from home production.

Of course, although induced comparative advantage can tilt the balance towards marriage it does not guarantee it. We have already seen that lack of companionship
or public good benefits from cohabitation work against marriage so strictly speaking, for \( \delta = 0 \) and \( \gamma = 0.5 \), the gains from comparative advantage would need to be strong enough to overcome this. In order to isolate this effect, we shall focus our attention on those cases where the male spouse brings sufficient market earnings to the match that the female will, all else equal, agree to the match. We therefore assume that the male specialises in market work and receives the maximum level of education.

Starting from this benchmark, as we shall show, the two factors that contribute to a rejection of marriage are (i) the male-female wage gap is such that the male might prefer to remain single rather than share his market income; \textit{and} (ii) there is a high subjective desire for education.\textsuperscript{20} This will be established by way of diagrams.

\textbf{[Figure 4 in here]}

In the upper panel of Figure 4, we have plotted agent \( X' \)'s utility, \( U_X \), against \( Y' \)'s education, \( e_Y \). If \( X \) remains single, his utility is depicted by the horizontal line \( U_X^s \), which is independent of \( e_Y \). If he marries, his utility is depicted by the curve \( U_X^m \) which rises and then declines in \( e_Y \), peaking at an optimal value labeled \( e_Y^m \). Note that \( e_Y^m \) is optimal from the male's point of view and does not coincide with the married female's own optimal level of education, which is denoted by \( e_Y^{m*} \), unless \( b = 0 \).

Five cases are possible: (i) that \( U_X^s \) lies above \( U_X^m \) for all feasible values of \( e_Y \); (ii) that \( U_X^s \) lies below \( U_X^m \) for all feasible values of \( e_Y \); (iii) that \( U_X^s \) lies above \( U_X^m \) at \( e_Y = 0 \) but below it at \( e_Y = \sigma \), (iv) that \( U_X^s \) lies below \( U_X^m \) at \( e_Y = 0 \) but above it at \( e_Y = \sigma \) and finally, (v) that \( U_X^s \) lies above \( U_X^m \) at both extreme values of \( e_Y \) but below it at intermediate values. The interesting cases are (iv) and (v). The diagram depicts the last case but note that this is qualitatively similar to case (iv).

\textsuperscript{20}By assumption, this affects both male and female equally but what counts is a high desire for education on the part of the female.
In this case, agent X will reject marriage with agent Y if her education level is either below a minimum value, labeled $\xi^m_Y$ or above a maximum, labeled $\bar{\xi}^m_Y$. We shall refer to these thresholds as marital desirability constraints, as outside them prospective spouse Y is considered as lacking either sufficient market or sufficient homemaking skill, or in the case of the lower threshold, possibly both.\footnote{For cases (iv) and (v) to arise, agent X should do relatively well by remaining single but not too well. Thus agent X’s own wage should be high enough but not so high relative to the benefits of getting married that he prefers to stay single regardless of the household services he can enjoy from a relatively skilled spouse. This combination is more likely to arise when there are large absolute differences in male and female wages.}

As we shall see below the minimum level of education that agent Y will undertake is $\hat{e}^m_Y$, so the lower marital desirability constraint never binds.

In the lower panel of Figure 4, we depict agent Y’s utility, $U_Y$, against her own education, $e_Y$. The broken lines labeled $U^s_Y$ depict her utility from remaining single while the solid lines labeled $U^m_Y$ depict it from getting married. Two cases are depicted: $b = 0$ and $b = b' > 0$. Parentheses (0) represent the first case and the parentheses ($b'$) represent the second. When $b = 0$, in the case of marriage, Y’s optimal education level, $e^*_m(0)$ will be equal to $\hat{e}^m_Y$, the level that X would ideally want Y to have. This is because when $b = 0$, both spouses fully internalise the benefits of each other’s education. Second, in the case depicted, Y’s optimal level of education when single, $\hat{e}^*_s(0)$, exceeds that when she marries $e^*_m(0)$ but the optimum utility obtained from getting married $U^*_m(0)$ exceeds that from staying single $U^*_s(0)$. This sets up a benchmark in which marriage is preferable to staying single for at least a very low value of $b$. Third, note that since, $e^*_m(0) < \bar{\xi}^m_Y$, agent Y’s optimal educational choice is not subject to a marital desirability constraint.

The second case depicted in the lower panel of Figure 4 is for $b'$ which is large enough that the upper marital desirability constraint (barely) binds. In this case, agent Y’s
(or her parents acting in her interest) her unconstrained optimum, $e^m_Y(b')$ coincides with the upper marital desirability constraint, $e^m_Y$. However, since $U^*_{ym}(b') > U^*_s(b')$ as shown, agent $Y$ prefers to get married to remaining single. We next show that as $b$ rises further, $U^*_s(b)$ rises faster than $U^*_y(b)$ so that for a high enough value of $b$, agent $Y$ will prefer to remain single rather than submit to a marital desirability constraint. This is shown in Figure 5 below.

In Figure 5, the solid lines continue to show agent $Y$’s utility while married and the broken lines her utility when single. When $b = b''$, agent $Y$’s married self’s utility increases only to the extent that at the constrained level of education, $e^m_Y$, her subjective utility from education goes up due to the increase in $b$. This is indicated by the double-headed arrow labelled $\Delta U^m_Y$. However, her single self’s utility increases both because of the increase in subjective utility at the initial level of education and because she will be able to increase her educational level in response. This increase is depicted by the arrow $\Delta U^s_Y$. As depicted, she is now indifferent between getting married or staying single. For a larger value of $b$, she would strictly prefer to remain single.

An observable implication of a subjective desire for education is that if it is strong enough, agent $Y$ will choose to remain single but by doing so, she will command a higher income and work more than if she had a weaker desire – what appears to be a pecuniary ambition would be driven by a non-pecuniary motive.\footnote{As $b$ crosses the critical value, $b''$ there will be a jump in educational level. Thus, there can be sharp differences in the education, marital status and work hours of a woman, depending on the underlying preference for education, even without any change in the underlying wage.}
On the basis of the foregoing analysis note that for any given configuration of other exogenous parameters, there are two different thresholds $b'$ and $b''$ which characterise how a female’s attitude to marriage depends on her underlying taste for education. For values of $b$ lying in the interval $[0, b']$ marriage does not impose any constraints on a female’s level of education; for $b$ in the interval $(b', b'')$ a female prefers to get married even though it means choosing a constrained level of education and for $b$ above $b''$ a female prefers to remain single in order to be free of constraints on her education.

The intermediate case is of interest because it suggests that when marriage is mutually consensual it can create a binding constraint than dominates other factors (such as the labor market value of education and the subjective preference for it) that affect parental decisions in this regard. A female whose subjective preference for education lies within a given can be said to face this constraint as she prioritises marriage over education. These results are captured in the following proposition.

**Proposition 3:** When $\omega_X > \omega_Y$, a female can face a binding constraint on her education, arising from her prospective spouse’s preferences. Three possible situations can arise, depending on the female’s subjective desire for education: for given value of other parameters there are two critical thresholds $b' > 0$ and $b'' > b'$ such that (1) a female with $b \leq b'$ is not bound by the constraint; (2) a female with $b \in (b', b'')$ submits to this binding constraint; (3) a female with $b \geq b''$ rejects the constraint and stays single.

How do the marital desirability constraints and the critical thresholds for $b$ change with the underlying wages and the degree of wage discrimination?
5.1 Reduction in wage discrimination:

We assume that a reduction in wage discrimination is implemented through increasing female wages while leaving male wages unchanged. The analysis of this case proceeds by appeal to Figures 4. If agent Y was doing some market work initially, an increase in female wages would shift $U^m_X$ outwards in Figure 4 while leaving $U^s_X$ unchanged (these shifts are not shown). This will lead to an increase in the upper marital desirability bound on female education. Thus marital desirability will be consistent with a higher level of education.

What does this do to the likelihood that, given a value of $b$, a female will be in case 1 (married and unconstrained in her level of education), 2 (married but unconstrained in her education) or 3 (unmarried and therefore unconstrained in her level of education) will depend on how the two thresholds $b'$ and $b''$ change as both a direct result of the increase in female wage and the indirect effect of an increase in the marital desirability constraint. In general, it is hard to pinpoint these changes.

In the special case where the married self of agent Y does no market work initially, a rise in female wages will have no effect on $U^m_X$ or $U^m_Y$. Hence neither $e^m_Y$ nor $b'$ change but since the single self would always work, $U^s_Y$ rises for every value of $b$ and this suggests that $b''$ falls. Thus the only effect is a greater likelihood that given $b$, a female will be in case 3 rather than in case 2. Thus a reduction in gender wage discrimination might make more women opt to remain single while not affecting the level of education of those that choose to get married (this of course, applies only to married women who specialise in housework).
5.2 Proportionate increase in wages:

We now consider a proportionate increase in wages at a constant degree of discrimination. Figure 6 below depicts an interesting possibility that might arise.

The top half of Figure 6 depicts agent $X$’s utility as a function of agent $Y$’s education; the second half depicts agent $Y$’s own utility as a function of her education. In the top half, a proportionate increase in wages is shown as shifting both $U_X$ and $U_X^m$ upwards but the shift in the former is somewhat greater than that in the latter. As a result, $\bar{e}_m^Y$ falls and the marital desirability constraint becomes even more binding.

Why is such an effect on $\bar{e}_m^Y$ plausible? Because agent $X$ is the principal breadwinner in the marital household, any increase in his wages is going to have be to shared at the margin with his spouse. Even though her own basic wage has gone up proportionately, her overall market income will increase less than proportionately so that the marginal income earned by agent $X$ will not accrue entirely to him. When single, both the absolute and the marginal subsidy are zero. Moreover, when married, agent $X$ will already be up against feasibility constraints in both education and market work, limiting the extra market income that he can earn in response to his higher wage. His single self, on the other hand, can always choose a higher level of education and more work hours. This is the context in which the above occurs.

In the bottom half of Figure 6, we show how the decrease in $\bar{e}_m^Y$ affects agent $Y$’s choices at different levels of her subjective preference for education. At the threshold value of $b'$, agent $Y$ was previously borderline-constrained by the upper bound on education but is now strictly constrained by the new one. This is because of two factors: (i) the fall in $\bar{e}_m^Y$ would induce this even if her married self does not work in
the labor market; (ii) if, in addition, her married self works, then the increase in \( \omega \)
will, increase \( e^m_Y(b') \). This is the case shown in Figure 6. The first factor is indicated
by the leftward arrow from the line indicating that initially, \( e^m_Y(b') = \bar{e}^m_Y \) and the
second by the rightward arrow from the same line.

Thus, given the tightening of the marital desirability constraint on female education,
agent \( Y \) becomes more likely to face a tradeoff between education and marriage, i.e.
to be in case 2 or 3 of Proposition 3, rather than in case 1. Which way this tradeoff
is resolved will depend on how close she initially was to the upper threshold \( b'' \) and
how that changes with the increase in wages.\(^{23}\) In the case depicted, she was initially
at the boundary between case 1 and case 2, and well inside the boundary between
case 2 and 3. Thus although her utility from remaining single jumps up substantially
from the increase in female wages, she nonetheless chooses marriage even though it
requires a lower level of education than at the old wage.

A case such as depicted in Figure 6 is likely to arise when female wages are initially
quite low, either because of high levels of discrimination or because of low labor
productivity in general. This would be associated with low levels of female labor force
participation so that a proportionate increase in wages would do little to increase the
female contribution to household income. In these circumstances, a male is likely to
become more demanding of household skills from his prospective partner.

Relatively poor economies, which combine low absolute wages with high degrees of
gender wage biases, are thus likely to witness a decline in female education and labor
force participation at the early stages of economic growth. This prediction is consis-
tent with empirical findings that female labor force participation first falls and then

\(^{23}\)With a tighter marital desirability constraint on female education, the benefits from remaining
single will be comparatively larger but this is balanced by the fact that as a married female, \( Y \) can
share in the higher income her spouse brings in as a result of the increase in his wages. The first
effect suggests that \( b'' \) will decrease while the second suggests that it will increase.
rises as a result of economic growth (see, e.g. Psacharopoulos and Tzannatos [1989]). This non-monotonicity is often attributed to changes in the composition of national income over the stages of growth. Agriculture, which is more conducive to female employment, declines while industry, which is less conducive at least initially, grows (Psacharopoulos and Tzannatos). The mechanism of our model is not inconsistent with the traditional explanation and could well shed light on why industry, in which employment is more formal than in agriculture and the required skills more dependent on formal education, is likely to be strengthened as the preserve of men during the early period of expansion.

Before concluding this section, two assumptions that merit further discussion are (i) the absence of a preference for leisure; (ii) the symmetry between agents.

Including a taste for leisure would not affect specialisation in the married household so long as the disutility of work was independent of the type of work undertaken, i.e. market work versus household chores. In that case, the leisure-work tradeoff would depend only on the total time in work, not how it was divided between market work and housework. Within the married household, maintaining the other assumptions such as ex-ante symmetry and unitary decision-making, each partner would spend the same amount of overall time in work but would continue specialising, either in a one-sided or mutual fashion, as in the benchmark model.

If the disutility of work was affected by the type of work, this would alter the nature of specialisation but not radically eliminate it. For example, if housework was more onerous than market work, it might be less likely for a female to specialise in housework, all else equal, but this might not change the result that a male specialises in market work, nor would it alter the biases induced by the marital division of labor between married females and their single selves.
The assumption of completely symmetry between genders is clearly unrealistic. It could well be argued that the reason why married women tend to specialise in housework is that they have an intrinsic advantage in some aspects of it, particularly in child-bearing and rearing. It is true that an absolute advantage on the part of either in gender in either aspect of the marital division of labor could explain this division without any need to invoke gender based wage inequality.

At the same time, to the extent that male-female productivity differences arise only because females have an absolute advantage in housework, not because males have an absolute advantage in market work, then in efficient and competitive labor markets we would expect to see gender wage equality. We would also see single male and females enjoying similar labor market outcomes. Thus both wage inequality and differences in time allocation for single persons are incompatible with any explanation that suggests women work less in labor markets because they are intrinsically better than men at taking care of the home.

6 Conclusions:

This paper has analysed a model of educational choice, time-use and marital choice in which labor market discrimination creates a wedge between male and female education and labor market outcomes and marriage exacerbates it. Our model suggests that neither would reducing labor market discrimination necessarily lead to a continuous decline in the male:female education gap nor would economic growth necessarily lead to a monotonic increase in the educational level of married females.

The analysis was positive in nature and not based on maximising social welfare functions. Nonetheless there were some insights into the welfare effects of wage discrimination. All else equal, wage inequality gives men an upper hand in the marriage
market. Because of their higher earning power, and consequently higher education, they are able to attract females into marriage at the expense of the latter’s own education and labor market participation.

Our paper has not questioned the source of labor market discrimination. However, models of statistical discrimination suggest that a circular relationship can develop between a disadvantaged group’s presence in the labor market and the tendency of employers to rationally rather than maliciously discriminate against them (see Aigner and Cain [1977] for an overview of such models). Our model suggests that marriage plays a compounding role in this relationship.


APPENDIX

The following results pertain to the case of gender wage discrimination, \( \omega_X > \omega_Y \).

**Lemma 1:** Under (A-4) and (A-5), the first-order conditions for both labor and education, and for both single and married couples imply that, given interior solutions

\[
\begin{align*}
\frac{\partial \ell_i^k}{\partial e_i^k} \bigg|_{e^*} & > 0 \\
\frac{\partial \ell_i^s}{\partial e_i^s} \bigg|_{e^*} & > 0
\end{align*}
\]

for \( i = X, Y \) and \( k = s, m; \mid e^* \) indicates a derivative along the first-order condition for labor while \( \mid_{e^*} \) indicates a derivative along the first-order condition for education.

**Proof:** Differentiating the relevant first-order conditions for \( \ell_i \) and \( e_i \), keeping in mind that the exercise applies only for interior solutions in the case of married agents, we get the following

\[
\begin{align*}
\frac{\partial \ell_i^s}{\partial e_i^s} \bigg|_{e^*} & = \frac{\left( u'' \cdot (\omega_i) (\alpha_i) (1 - \ell_i^s) + h' \right)}{\left( u'' \cdot (\omega_i)^2 (e_i^s)^2 + \alpha^2 h'' \right)} \\
\frac{\partial \ell_i^s}{\partial e_i^s} \bigg|_{e^*} & = \frac{\left( u'' \cdot (\omega_i)^2 + (\alpha_i')^2 (1 - \ell_i^s)^2 h'' + \alpha'' h' \cdot (1 - \ell_i^s) \right)}{\left( u'' \cdot (\omega_i)^2 (e_i^s)^2 + \alpha^2 h'' \right)} \\
\frac{\partial \ell_i^m}{\partial e_i^m} \bigg|_{e^*} & = \frac{\left( u'' \cdot (\omega_i) (\alpha_i) (1 - \ell_i^m) + h' \right)}{\left( u'' \cdot (\omega_i)^2 (e_i^m)^2 + \alpha^2 h'' \right)} \\
\frac{\partial \ell_i^m}{\partial e_i^m} \bigg|_{e^*} & = \frac{\left( u'' \cdot (\omega_i)^2 + (\alpha_i')^2 (1 - \ell_i^m)^2 h'' + \alpha'' h' \cdot (1 - \ell_i^m) \right)}{\left( u'' \cdot (\omega_i)^2 (e_i^m)^2 + \alpha^2 h'' \right)}
\end{align*}
\]

where \( j \) indicates a variable related to the spouse in the married agent’s case. There are only two cases where this is relevant: (i) \( i = Y \) and \( j = X \) when \( \omega_X e_X \ell_X = \omega_X \sigma \); (ii) \( i = X \) and \( j = Y \) when \( \omega_Y e_Y \ell_Y = 0 \).

In all cases, (A-4) and (A-5) are sufficient for the above to all be positive.

**Lemma 2:** Concavity of the maximisation problem requires that

\[
\frac{\partial \ell_i^k}{\partial e_i^k} \bigg|_{e^*} > \frac{\partial \ell_i^k}{\partial e_i^k} \bigg|_{e^*}
\]

for \( i = X, Y \) and \( k = s, m \).

**Proof:** For single agents, the Hessian matrix formed by the own- and the cross-partial derivatives of each first-order condition with respect to each choice variable is two-dimensional. For married agents, there are four endogenous variables in principle. However, because at least one spouse is always in a corner with respect to labor supply, the Hessian is also two-dimensional, if it is defined at all.
The determinant of the Hessian is equal to:
\[
\left. \frac{\partial^k \ell_i}{\partial e_i^k} \right|_{e^*} - \left. \frac{\partial^k \ell_i}{\partial e_i^k} \right|_{\ell^*}
\]
where, in the married agent’s case, \(i\) is the spouse, if any, whose labor supply is interior.

It is easily verified that the diagonal elements of the Hessian are negative so the principal minors alternate in sign as (required for concavity) if and only if the above expression is positive.

**Lemma 3:** Under (A-4),
\[
\left. \frac{\partial^k \ell_i}{\partial \omega_i^k} \right|_{\ell} > 0 \quad \left. \frac{\partial^k e_i}{\partial \omega_i^k} \right|_{e} > 0
\]
where \(k = s, m\) and \(i = X, Y\).

**Proof:** We show this for a single agent. The case of a married agent (assuming an interior solution) follows analogously.

Taking the derivative of the first-order condition for labor supply of a single agent with respect to \(\ell_i^s\) and \(\omega_i\), solve for:
\[
\left. \frac{\partial \ell_i^s}{\partial \omega_i} \right|_{\ell} = \frac{-[u''e_i^s + u']}{u''(\omega_i)^2(e_i^s)^2 + \alpha^2h''} > 0 \quad \text{by (A-4)}
\]

Taking the derivative of the first-order condition for education of a single agent with respect to \(e_i^s\) and \(\omega_i\), solve for
\[
\left. \frac{\partial e_i}{\partial \omega_i} \right|_{e} = \frac{-[u''e_i^s + u']}{u''(\omega_i)^2 + (\alpha')^2(1 - \ell_i^s)^2h'' + \alpha'h'(1 - \ell_i^s)} > 0 \quad \text{by (A-4)}
\]

**Lemma 4:** For a given value of \(e_Y^s\), \(\ell_Y^m < \ell_Y^s\).

**Proof:** This is obvious when \(\ell_Y^m = 0\) so the only case in which it needs to be proven is when \(\ell_Y^m > 0\). Note that if this is the case then \(e_X = \sigma\) and \(1 - \ell_X = 0\).

Now suppose that in fact \(\ell_Y^m \geq \ell_Y^s\). Then \(1 - \ell_Y^s \geq 1 - \ell_Y^m\). By the concavity of \(h(\cdot)\) and the fact that \(2\gamma \geq 1\), it must then be the case that \(2\gamma \alpha_Y h'(\alpha_Y(1 - \ell_Y^m)) \geq \alpha_Y h'((\alpha_Y(1 - \ell_Y^s)))\). From the respective first-order conditions for the labor supply of agent \(Y\)’s married and single self (both of which hold as equalities in this case), it follows from comparing terms that
\[
u'(w_X \sigma + w_Y e_Y \ell_Y^m) \geq u'(w_Y e_Y \ell_Y^s)
\]
By the concavity of $u(\cdot)$, this implies that

$$\frac{w_X\sigma + w_Ye_Y\ell^m_Y}{2} \leq w_Ye_Y\ell^s_Y.$$  

But since our working hypothesis is that $w_Ye_Y\ell^m_Y \geq w_Ye_Y\ell^s_Y$, the above can only be true if $w_X\sigma \leq w_Ye_Y\ell^s_Y$, which is not possible since $w_X > w_Y$ and $\sigma \geq e^s_Y$. Thus $\ell^m_Y < \ell^s_Y$.

**Lemma 5:** Suppose $b = 0$. At given $\ell_Y$, for any value of $\gamma \in [0.5, 1]$, $(i)e^m_Y < e^s_Y$.

**Proof:**

Recall equation (8). Taking $b = 0$, this can be written for agent $Y$'s single self as

$$u'(\omega_Ye^s_Y\ell^s_Y)\omega_Y\ell^s_Y = -h'(\alpha(e^s_Y)(1 - \ell^s_Y))(1 - \ell^s_Y)\alpha'(e^s_Y)$$

Similarly recall equation (10). Taking $b = 0$ and noting that in the relevant case, $e^m_X = \sigma$ and $\ell^m_X = 1$ it can be rewritten for the married self of agent $Y$ as

$$u'(\frac{e^m_X + e^m_Y}{2})\omega_Y\ell^m_Y = -2\gamma h'(\alpha(e^m_Y)(1 - \ell^m_Y))(1 - \ell^m_Y)\alpha'(e^m_Y)$$

Now let $\Lambda = -2\gamma h'(\alpha(e)(1 - \ell))(1 - \ell)\alpha'(e)$ for a generic agent. Differentiating $\Lambda$ with respect to $e$:

$$\frac{\partial \Lambda}{\partial e} = -2\gamma h''(\cdot)(1 - \ell)^2(\alpha(e))^2 - 2\gamma h'(\cdot)(1 - \ell)\alpha''(e) > 0$$

since both $h''$ and $\alpha''$ are negative. Also,

$$\frac{\partial \Lambda}{\partial \gamma} = -2\gamma h'(\cdot)(1 - \ell)\alpha'(e) > 0$$

since $\alpha' < 0$ at the optimum.

Now let $e^s_Y$ solve equation (8) and evaluate equation (10) at this value of $e^s_Y$. Then the LHS of equation (8) must be strictly greater than that of equation (10) since, at $e^s_Y$ and for any given value of $\ell_Y$, agent $Y$’s claim on market income will be higher when married to the higher paid agent $X$ than when single.

Furthermore, the RHS of equation (8) must be less than or equal to that of equation (10) since the two will be equal when $\gamma = 0.5$ and the RHS of equation (10) will be strictly greater than that of equation (8) when $\gamma > 0.5$.

Hence, evaluated at $e^s_Y$, the marginal cost of education for prospectively married agent $Y$ will be strictly greater than the marginal benefit; implying that $e^m_Y < e^s_Y$ for given values of $\ell^Y$.

**Lemma 6:** When $\omega_X = \omega_Y = \omega$, $\gamma = 0.5$, $\delta = 0$, then $U^s_i \geq U^m_i$, $i = X, Y$.  

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Proof: Note that in the symmetric case, a each agent’s single self makes identical choices while it is easy to show that married couples also make identical choices. Thus \( \ell^s_X = \ell^s_Y = \ell^s, e^s_X = e^s_Y = e^s \) and using similar notation for \( \ell^m_X, \ell^m_Y \):

\[
U^s(\ell^s, e^s) = u(\omega e^s \ell^s) + h(\alpha(e^s)(1 - \ell^s)) \\
\geq u(\omega e^m \ell^m) + h(\alpha(e^m)(1 - \ell^m)) \\
\geq u(\omega e^m \ell^m) + 0.5h(2\alpha(e^m)(1 - \ell^m)) = U^m(\ell^m, e^m)
\]

where the first inequality is due to revealed preference and the second due to the concavity of \( h \).
Figure 2
Figure 4
Figure 5
Figure 6