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ABSTRACT

Il SIR (suscettibile-infetto-recuperato) è un modello epidemiologico a componenti che si escludono vicendevolmente. In questo lavoro l'obiettivo è quello di modellizzare un rischio per le PMI (piccole e medie imprese) appartenenti a settori specifici o a determinate filiere produttive, in cui più agenti interagiscono tra loro e generano rapporti di contagio: proponiamo a tal fine un particolare modello SIR di rischio finanziario.

SIR (susceptible-infected-recovered) is an epidemiological model given by mutually exclusive components. In this work, the aim is to model a risk for the SMEs (small and medium-sized enterprises) operating in specific supply chains, in which multiple agents interact with each other and generate contagion relationships: for this purpose, we propose a particular SIR model of financial risk.

PAROLE CHIAVE

Covid-19 – Rischio contagio – Crisi finanziaria – PMI.

Covid-19 – Contagion risk – Financial crisis – SME.

1. COVID-19 is spreading human suffering worldwide: that is what we should all be focused on. But we are not doctors. We are economists, and COVID-19 is most definitely spreading economic[1] suffering worldwide. The virus may in fact be as contagious economically as it is medically. The transmission channels that transform the health emergency in an economic crisis are complex because they include both direct and indirect effects: to social distancing measures correspond negative effects on both, supply and demand for goods and services, and on the investment choices of companies, with an inevitable impact on the financial system. In this context the pandemic has particularly affected the most fragile organisations, such as small and

* Saggio sottoposto a referaggio secondo il sistema di *peer review*.

[1] R. Baldwin, B. Weder di Mauro (2020). *Economics in the time of COVID-19*. CEPR Press; J. W. Goodell (2020). *COVID-19 and finance: Agendas for future research, Finance Research Letters*, Elsevier, 35.

medium-sized enterprises (SME)[2], typically characterised by a lack of strategic, financial, and technological resources. Not all SMEs, however, have been equally affected by the health emergency: in some industries, the consequences have been considerable, due to health precautions; companies in other economic sectors, however, have reacted, by investing in sustainability and digitalisation[3], reviewing their business models[4], also thanks to the use of public funds.

While the literature on the contagion in financial sector is sufficiently developed, we cannot say the same on that between non-financial companies [5]. However, some theoretical arguments, valid for the financial system, can be extended to non-financial companies[6].

It is necessary to point out that, unlike the financial systems, the relationships between companies are often long-term, based on technological specificity and on stable supply chains. Indeed, the presence of phenomena of a relational nature, that characterizes the analysis of the contagion, hold that the traditional methodologies - referring to independent observations - is unable to grasp the relational nature of economic phenomena.[7]

Given the above considerations, arises the need to develop an epidemiological model that consider the complexity of the supply chain relationships in network of non-financial companies, with particular reference to SMEs.

In order to address the gap in the literature, this paper aims to study the basic reproduction number R_0 of companies due to the COVID-19 pandemic, with reference to SMEs operating in specific supply chains. R_0 is an indication of the transmissibility of a failure, representing the average number of new failures generated by a failure of a company in the system. For $R_0 > 1$, the number of failures is likely to increase, and for $R_0 < 1$, transmission is likely to die out.

[2] SMEs are defined as enterprises with fewer than 250 employees, whose annual turnover does not exceed 50 million euros or whose annual balance sheet total does not exceed 43 million euros. European Union User's Guide to the definition of SMEs, Publications Office of the European Union., Luxembourg, 2015.

[3] T. Pencarelli, F. M. Cesaroni, P. Demartini (2020). *Covid-19 and Italian small and medium-sized enterprises: consequences, threats and opportunities*, Piccola Impresa / Small Business, n. 3, p. 10; Rapporto Cerved PMI 2020, novembre 2021.

[4] A. Di Vaio, F. Boccia, L. Landriani, R. Palladino (2020). *Artificial intelligence in the agri-food system: Rethinking sustainable business models in the COVID-19 scenario*. Sustainability, Vol. *12*, N. 12, 4851; C. Bagnoli, S. Biazzo, G. Biotto, M. Civiero, A. Cuccu, G. P. Lazzer, M. Massaro, M. C. Pignata, M. Renosto (2020). *Business Models Beyond Covid-19 50+1 paradossi da affrontare per l'efficace gestione strategica di una crisi*, SIH - Strategy Innovation Hub, pp. 1-46.

[5] J. Chevallier, (2020). COVID-19 pandemic and financial contagion. Journal of Risk and Financial Management, 13(12), 309; R. M. May, S. A. Levin, G. Sugihara (2008). *Complex Systems: Ecology for Bankers*. Nature, 15(51):893-895.

[6] M. Akhtaruzzaman,, S. Boubaker,, A. Sensoy,(2021). Financial contagion during COVID–19 crisis. *Finance Research Letters*, *38*, 101604.Y. Leitner (2005). *Financial Networks: Contagion, Commitment and Private Sector Bailouts*. Journal of Finance, 60: 2925-2953.

[7] L. Biggiero (ed) (2016). *Relational Methodologies and Epistemology in Economics and Management Sciences.* IGI Global.

The basic reproduction number is a relevant indicator of a company bankruptcy-risk assessment, indicating the crisis risk of a company with respect to the epidemic spread.

2. In this section we introduce the model. The starting point is a discrete-time version of a standard compartmental epidemiological SIR (Susceptible-Infectious-Recovered) model[8]: it can be used to describe the spread of a disease within a population[9]. It is a dynamical system that describes the time evolution of the following three populations: susceptible individuals $S(t)$, infected individuals $I(t)$ and removed individuals $R(t)$. SIR model is the basis of deterministic and stochastic-probabilistic models.

In this work we turn the SIR model to economy, aiming to evaluate the company failure process as an infection process. In particular, we analyze how these companies became "contagious". The SIR model assumes a fixed population size of N and the variables $S(t)$, $I(t)$ and $R(t)$ denote the number of companies in the three groups described below, as functions of time t.

$S(t)$: the number of susceptible companies at time t (all companies that are not in crisis but could be infected);

$I(t)$: the number of infected companies at time t (companies in crisis that could infect others);

$R(t)$: the number of recovered or resistant companies at time t (companies in crisis that have been able to reinvent themselves and therefore have emerged from the crisis) and/or the number of deaths at time t. (All the companies that have gone bankrupt and therefore have exited the market).

Let N be the number of companies. We can define:

$s(t) = \frac{1}{\mathrm{N}} S(t)$: the fraction of susceptible companies at time t (1)

$i(t) = \frac{1}{\mathrm{N}} I(t)$: the fraction of infected companies at time t (2)

$r(t) = \frac{1}{\mathrm{N}} R(t)$: the fraction of recovered or resistant companies at time t. (3)

[8] Q. Li, X. Guan, P. Wu, X. Wang, L. Zhou, Y. Tong, R. Ren, K.S.M. Leung, E.H.Y. Lau, J.Y. Wong (2020). *Early transmission dynamics in Wuhan, China, of novel coronavirus–infected pneumonia*. New England Journal of Medicine.

[9] W.O. Kermack and A.G. McKendrick (1927). *A Contribution to the Mathematical Theory of Epidemics*. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 115, pp. 700– 721. https://doi.org/10.1098/rspa.1927.0118.

Neglecting new-born and deceased companies for other causes, we have $N = S + I + R$. Thus, we must compute only two variables since the third can be derived from that relation.

Let β be the average number of appropriate contacts (useful for the transmission of the corporate crisis) of a company in the unit of time and γ be a recovery rate due to State intervention. Thus, we have:

$$\frac{\beta\, I(t)}{N} = i(t) \qquad (4)$$

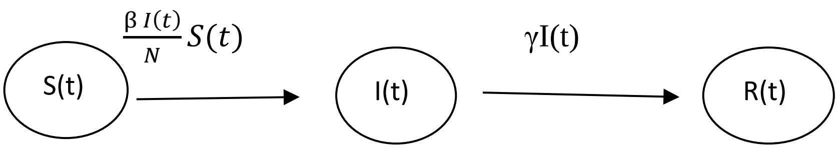
Where $i(t)$ is the average number of appropriate contacts of a susceptible company with infective companies in the unit of time. Thus, the number of new infective (that is, of susceptible that become infectious) in the unit of time is:

$$\frac{\beta\, I(t)}{N} S(t) = \beta N s(t) i(t) \qquad (5)$$

In a similar way we can define the number of companies that have been removed in the unit of time as:

$$\gamma I(t)$$

We can think of compartments as tanks from which firms (companies) can enter or exit due to their change of status (from Susceptible to Infectious and from Infectious to removed).



Therefore, we can write the SIR model using ordinary differential equations (ODEs), which implies a deterministic model with continuous time (as opposed to discrete time). The SIR Model is described by the following differential equations:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\dfrac{\beta I(t)}{N} S(t) \\ \dfrac{dI(t)}{dt} = +\dfrac{\beta I(t)}{N} S(t) - \gamma I(t) \\ \dfrac{dR(t)}{dt} = +\gamma I(t) \end{cases} \qquad (6)$$

Based on these assumptions the rates of change of the three populations are described by the following system. Constituted by three differential equations satisfying $N = S(t) + I(t) + R(t)$. Thus, a consistent model must verify the following equation:

$$\frac{dS(t)}{dt} + \frac{dR(t)}{dt} + \frac{dI(t)}{dt} = 0 \tag{7}$$

Finally, it should be noted that β and γ have the inverse dimension of time.

We observe that dividing both sides of the equations (3) by N you get:

$$\begin{cases} \frac{dS(t)}{dt}\frac{1}{N} = -\frac{\beta I(t)}{N}S(t)\frac{1}{N} \\ \frac{dI(t)}{dt}\frac{1}{N} = +\frac{\beta I(t)}{N}S(t)\frac{1}{N} - \gamma I(t)\frac{1}{N} \\ \frac{dR(t)}{dt}\frac{1}{N} = +\gamma I(t)\frac{1}{N} \end{cases} \tag{8}$$

Hence by substituting equation 1,2 and 3 in 9 we get:

$$\begin{cases} \frac{ds(t)}{dt} = -\beta i(t)s(t) \\ \frac{di(t)}{dt} = +\beta i(t)s(t) - \gamma i(t) \\ \frac{dr(t)}{dt} = +\gamma i(t) \end{cases} \tag{9}$$

Variables $s(t), i(t), r(t)$are all between 0 and 1.
Finally, it should be noted that the third equation can be omitted because its dynamics is decoupled from the dynamics of the other two variables and it can be derived from the other two variables, via the expression $r(t) = 1 - s(t) - i(t)$.
If we denote:

$$x_1(t) = s(t)$$
$$x_2(t) = i(t)$$
$$y(t) = r(t)$$

the non-linear system is obtained:

$$\begin{cases} \dot{x}_1 = -\beta x_1(t)x_2(t) \\ \dot{x}_2 = -\gamma x_2(t) + \beta x_1(t)x_2(t) \\ y(t) = 1 - x_1(t) - x_2(t) \end{cases} \tag{10}$$

Compartmentalized models are characterized by the definition of some typical parameters:

- R_0 (basic reproduction number) defined as the average number of infectious companies produced by a single infectious company introduced into a fully susceptible population. It is defined only at the beginning of the infection.
- R (replacement number) is the average number of infectious subjects produced by each infectious at any time of infection.
- σ (contact number) defined as the average number of appropriate contacts of an infective at any time of infection.

Furthermore, we suppose that $R = R(t)$, $R(t) < R_0$, σ constant.
At the beginning of the infection, we have that $R(0) = \sigma\, s(0)$. If $s(0) = 1$ that is the initial companies are completely susceptible, then $R(0) = \sigma$, but $R(0) = R_0$ and so $R(0) = R_0 = \sigma$.

We know that $\sigma = \frac{\beta}{\gamma}$, that is the average number of contacts of an infected company during the crisis σ is obtained by multiplying the average number of a company called in the unit of time by the average time (β) for which the company is infected (crisis) (γ).
If we suppose that the initial company is completely susceptible, we have $R_0 = \sigma$ and thus $R_0 = \frac{\beta}{\gamma}$.

From (9) dividing the first equation by the third we obtain:

$$\frac{ds}{dr} = \frac{-\beta i(t)s(t)}{\gamma i(t)} \tag{10}$$

and then

$$\frac{-\beta i(t)s(t)}{\gamma i(t)} = -R_0 s(t) \tag{11}$$

from which

$$\frac{ds}{s} = -\, R_0 dr \tag{12}$$

Integrating both members of (12) we have

$$\ln(s)_{s(0)}^{s(t)} = -R_0[r(t) - r(0)] \tag{13}$$

$$ln\frac{s(t)}{s(0)} = -R_0[r(t) - r(0)] \tag{14}$$

$$s(t) = s(0)e^{-R_0[r(t)-r(0)]}$$

We can deduce that the larger R_0 the more rapidly the crisis spreads among companies. Precisely, the number of susceptible companies decrease, that is, they become infected, therefore the number of infected companies increase, and the removed ones continue to increase.

3. One of the issues related to the proposed model will concern the identifiability of the problem and the technique to solve this system of differential equations to obtain estimates of the parameters β and γ. Severals methods[10] could be used to fit the SIR model[11], including least squares, maximum likelihood, and a nonlinear dynamic one, all to be tested.

4. At the moment we have only theorized the dynamic behavior of contagiousness, that we can easily refer to SMEs in a specific supply chain, based on a SIR epidemic model. An optimal control problem has been proposed, in order to reduce the number of contagious companies, to avoid serious economic consequences.

[10] K. McAlinn and M. West (2019). *Dynamic Bayesian predictive synthesis in time series forecasting*. Journal of Econometrics, 210, pp. 155–169. https://doi.org/10.1016/j.jeconom.2018.11.010.
[11] J. Chu (2021). *A statistical analysis of the novel coronavirus (COVID-19) in Italy and Spain*, Plos one.