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Evaluating patterns of income growth when status matters: a robust approach

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Abstract

This paper addresses the problem of ranking growth episodes from a microeconomic perspective. While most of the existing criteria, framed in the pro-poor growth tradition, are either based on anonymous individuals or use to identify them on the base of their status in the initial period, this paper proposes new criteria to evaluate growth, which are robust to the choice of the reference period used to identify individuals. Suitable dominance conditions that can be used to rank alternative growth processes are derived by means of an axiomatic approach. Moreover, the theoretical results are used to rank the different growth episodes that took place in the last decade in Australia, Germany, Korea, Switzerland, and US.

Keywords: growth, income mobility, inequality, social welfare, pro-poorness.

JEL codes: D31, D63, D71.

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1 Introduction

Since the pioneering work of Kuznets (1955), the analysis of the distributional implications of growth has become one of the most prominent topics in economics. However, the impressive number of contributions that soon followed were not able to reach a unanimous consensus on the effects of growth on the distribution. The scarcity of reliable data and the use of aggregate indicators of the distribution and its dynamics were among the main reasons for this lack of consensus (see Ferreira, 2010). After a period of reticence, this issue is now living a renewed and increasing interest among scientists and policymakers. The availability of better survey data has spurred the scientific community to adopt a different perspective to this analysis. There is, in fact, an increasing awareness in the recent literature that individuals, rather than a representative aggregate of the whole population, should be the focus of analysis for evaluating the impact of growth (see, among others, Benjamin et al., 2011; Ravallion, 1998, 2012; Ravallion and Chen, 2007). Moreover, this aspect of growth is at center stage also in the political agenda at an international level: for instance, one of the targets of the Sustainable Development Goals (SDGs), that from the end of 2015 will replace the Millennium Development Goals, is to promote 'inclusive economic growth', that is growth that benefits all the segments of society.

Hence, as a response to the original macroeconomic approach, two alternative microeconomic approaches have been developed to evaluate growth and its distributional implications. The first is the disaggregated anonymous approach, which evaluates growth processes on the basis of the income change experienced by each part of the distribution (see, among others, Essama-Nssah, 2005; Ravallion and Chen, 2003; Son, 2004). Its main tool is the Growth Incidence Curve (GIC, Ravallion and Chen, 2003), plotting against each percentile of the distribution the mean income change of that percentile. This approach has however been recently criticized since, due to its anonymity properties, incomes of different individuals are used to compute the percentile specific growth, as those that are at a particular percentile in the initial income distribution are not necessarily at that same percentile in the second period income distribution. Measures of anonymous growth, thus, fail to capture the impact of re-ranking or mobility taking place during the overall growth process. This procedure can be satisfactory if the aim is the understanding of the pure cross-section impact of growth. In can be however undesirable if the aim is a more normative and intertemporal evaluation of growth.

In order to overcome this issue, a non-anonymous approach has been recently proposed, which relaxes the anonymity assumption and evaluates growth processes on the basis of individuals' growth experiences and their identities (see, among others, Bourguignon, 2011; Dhongde and Silber, 2015; Grimm, 2007; Jenkins and Van Kerm, 2006; Palmisano and Peragine, 2015; Palmisano and Van de gaer, 2013). In this approach, the identity of individuals is defined on the base of their status, namely their position in the initial distribution of income. Its main tool is the non-anonymous

Growth Incidence Curve (na-GIC, Bourguignon, 2011; Grimm, 2007; Van Kerm, 2009), which plots the change in mean income of those individuals belonging to the same quantile in the initial distribution of income as a function of their quantile in this initial distribution. This approach is also called history dependent. The related frameworks usually give more weight to the growth experienced by the initially poor than to that experienced by the initially rich individuals.

In this article, we share this view and we believe that, for the welfare evaluation of growth, the status of individuals do matter. This information allows to find out who are the winners and losers from growth, a useful information, for example, in the evaluation of the efficacy of policy reforms, usually hidden by the anonymity assumption in the standard anonymous approach.

However, the existing literature has developed dominance conditions in which the identification of the individuals is exclusively based on their position in the income distribution of the first period (see in particular, Bourguignon, 2011; Palmisano and Peragine, 2015; Van Kerm, 2009). This choice, though perfectly legitimate, is not the only possible and is not supported by any normative grounded reason. Nevertheless, the choice of the reference period can have an impact on the result of a given comparison between countries or between growth episodes for the same countries. In other words, it is questionable, in the social evaluation of growth, to give priority only to the growth of the initially poor individuals, as compared to the initially rich, and not, for instance, to the growth of the finally poor individuals as compared to the finally rich.

Note that, in such frameworks, the attention on the poorer individuals in the final period also finds its justification in the sphere of public interventions that should, in general, target this group of the population. In fact, in terms of priority in public interventions (such as anti-poverty policies), it may be more meaningful to give relevance to the new poor rather than to those individuals who exited from poverty.

The considerations above are even more impelling when the evaluation of the distributional effect of growth concerns growth processes that take place over a long period or even over the full life span. In this case, the choice of the first period distribution as the reference distribution can be justified by the belief that low income earlier in life may impact the standards of living later in life but not the other way around. However, it does not imply that this belief will be universally accepted. There can be arguments in favor of the opposite belief, that is living with a low level of income may matter more the closer an individual is to the end of his/her life (see on this Hoy and Zheng, 2011).

In order to address this issue, this paper proposes a more general framework for the normative assessment and comparison of growth processes: within this normative framework, we are able to obtain dominance conditions that are robust with respect to the choice of the reference period used to identify individuals.

More in particular, we propose a social evaluation function (SEF) in which the status of the individuals both in the initial *and* in the final period can be used to evaluate growth. We, then, introduce desirable properties that allow to consider classes of this SEF, within which the two periods equally affect the social evaluation of growth, and classes of this SEF within which the first period status matters more or less than the final period status. By demanding for unanimity within these classes, we obtain distributional criteria to rank growth processes that result to be robust to the choice of the reference period used to identify individuals.

Hence, we provide new partial orderings for ranking growth processes, that are based on the concept of upward dominance for continuous distribution and upward and downward dominance for discrete distributions. Thus, our framework represents an additional instrument in the researcher's toolbox to help apprehending the distributional effects of growth, in particular when there is interest in making comparisons of growth between two (or more) populations or over time.

Last, note that our framework is also coherent with that part of the economic literature, mostly focused on happiness studies, in which increasing evidence is provided showing that individual well-being strictly depends on their income relative to that of the others (see on this Clark et al., 2008). In fact, in our framework the evaluation of a person's income growth implicitly depends on the incomes of other individuals in the population.

We, then, adopt this theoretical framework to compare the distributional impact of growth in five different countries, namely Australia, Germany, Korea, Switzerland, and US, in the last decade. We do this using the Cross National Equivalent File (CNEF), a dataset containing harmonized data on these countries. We find that Australia, followed by Korea, arises to be the best performing country, that is its growth process, evaluated when both initial and final period status matter, results to be the dominating process in the largest number of pairwise comparisons. Whereas, Germany and Switzerland arise to be the worst performing, that is, their growth processes results to be the dominated one in most of the pairwise comparisons considered.

Our results also show that it does make a difference in the ranking of countries whether one is concerned with the initial status of individuals or with their status in the final distribution. Thus it shows the relevance of adopting our generalized framework, which is able to provide additional information for the comparison of different growth processes.

The rest of this paper is organized as follows. Section 2 introduces the models used in the microeconomic-oriented literature on the distributional effect of growth and proposes the new framework. Section 3 provides the empirical analysis. Section 4 concludes.

2 Evaluating patterns of income growth

In this section we outline the set up and the standard tools used to assess alternative growth patterns. We then introduce our approach based on an extended concept of non-anonymity, that is robust to the choice of the reference period.

2.1 Standard practices

Let a society's income distribution be represented by the cumulative distribution function (*cdf*) $F : \Re_+ \to [0,1]$. In a given period of time t, $F(y_t) = P(\tilde{y}_t \in \Re_+ : \tilde{y}_t \leq y_t)$, that is the *cdf* returns the probability $p \in [0,1]$ of observing income less or equal to $\tilde{y}_t$ in that society in period t. The mean income of this society is denoted by $\mu(y_t)$. Let the inverse of this *cdf* be denoted by $y_t(p_t)$, where $y_t(p_t) = inf\,\{y_t \in \Re_+ | F(y_t) \geq p_t\}$; hence, $y_t(p_t) : [0,1] \to \Re_+$ represents the income of the person whose rank in the distribution $F(y_t)$ is p_t. p_t then represents the status of the individual in t. We deal with a total number of periods equal to 2, with $t = 1$ representing the pre-growth period, while $t + 1 = 2$ representing the post-growth period, hence $t \in \{1, 2\}$.

The standard anonymous practice to evaluate and compare the distributive performance of two growth processes consists in comparing their respective GICs and cumulative GICs. The GIC is formally defined as follows (Ravallion and Chen, 2003):

$$g\left(p\right) = \frac{y_{t+1}\left(p_{t+1}\right)}{y_t\left(p_t\right)} - 1 = \frac{L'_{t+1}\left(p_{t+1}\right)}{L'_t\left(p_t\right)}\left(\gamma + 1\right) - 1, for\ all\ p \in [0,1] \tag{1}$$

where $L'\left(p\right)$ is the first derivative of the Lorenz curve at percentile p, and $\gamma = \mu\left(y_{t+1}\right)/\mu\left(y_t\right) - 1$ is the overall mean income growth rate. The GIC plots the percentile specific rate of income growth in a given period of time. Clearly, $g\left(p\right) \geq 0$ ($g\left(p\right) < 0$) indicates a positive (negative) growth at p. A downward sloping GIC indicates that growth contributes to equalize the distribution of income (i.e. $g\left(p\right)$ decreases as p increases), whereas an upward sloping GIC indicates a non-equalizing growth (i.e. $g\left(p\right)$ increases as p increases). When the GIC is an horizontal line, inequality does not change over time and the rate of growth experienced by each quantile is equal to the rate of growth in the overall mean income.

Given two growth processes A and B, dominance of A over B is verified when the GIC of the former lies nowhere below that of the latter, in which case it is possible to state that under A all income percentiles have been growing more (or decreasing less) than under B. A dominance of the second order of A over B is verified when the cumulative GIC of the former lies nowhere below that of the latter, implying that A has been more progressive than B.

These criteria are based on the comparison of each income percentile at two different points in time. Therefore, although based on individual data, this procedure ignores the individuals' identity and does not allow to trace their income dynamic. It is then necessary to resort to the non-anonymous versions of the GIC, namely the na-GIC and cumulative na-GIC, in order to address this issue. Letting $y_{t+1}(p_t)$ be the final period income of an individual ranked p_t in the initial period, the na-GIC can be formally defined as follows (Bourguignon, 2011; Grimm, 2007; Van Kerm, 2009):

$$g\left(p_t\right) = \frac{y_{t+1}\left(p_t\right)}{y_t\left(p_t\right)} - 1, for\ all\ p_t \in [0,1] \tag{2}$$

In other words, the na-GIC associates to every quantile of the initial distribution the mean income growth of all individual units in that quantile.[1] In the same vein as the anonymous approach, dominance of process A over B is verified when the na-GIC of the former lies nowhere below that of the latter. In this case it is possible to state that under A the income of the individuals in each initial percentiles grows more (or decreases less) than under B. A dominance of the second order of A over B is verified when the cumulative na-GIC of the former lies nowhere that of the latter, implying that A favors more than B the income growth of the initially poor as compared to that of the initially rich.

Some recent contributions in the literature propose normative characterizations for the non-anonymous approach and hence provide a normative justification for the use of the na-GIC. In particular, they propose to evaluate growth episodes by means of a social evaluation function, which is assumed to be a function of the individuals' income change and the individuals' position in the initial distribution (see Bourguignion, 2011; Jenkins and Van Kerm, 2011; Palmisano and Peragine, 2015).

A major drawback of these frameworks, however, is their dependency on the first period distribution, as they are *sensitive* to the status of individual in the initial period but not to the status of individuals in the final period (unless no reranking takes place). Although the choice of the first period as the reference period to identify individuals is usually considered to be a natural choice, it still remains a purely arbitrary modeling choice. As discussed in the first section, different considerations do motivate a generalization of these frameworks to allow for a more flexible assessment of the distributional impact of growth. We do this in the following section.

2.2 The model

Our aim is to evaluate and compare growth processes according to an extended non-anonymous perspective. Therefore, we need to keep track of the status of individuals in both periods, where such status is represented by the rank of individuals in the initial and final distribution of income. For this reason, we denote by $\delta(p_t)$ the income change in moving from date t to $t+1$, for a given individual ranked p in the initial period and by $\delta(p_{t+1})$ the income change in moving from date t to $t+1$, for a given individual ranked p in the final period.

We denote by $G^{(t,t+1)}$ the growth process taking place between t and $t+1$ and by D the set of admissible growth processes. We are interested in ranking members of D from a normative perspective and we assume that such ranking can be represented by a social evaluation function, $\hat{W} : D \to R$. On the base of the arguments outlined so far, we propose that social preferences over growth processes can be represented by the following social evaluation function, which is a

[1] Note that the na-GIC is equivalent to a specific type of the Mobility Profile in Van Kerm (2009).

generalization of the rank dependent SEF proposed by Yaari: (1988)[2]

$$\hat{W}(G^{(t,t+1)}) = \frac{1}{2}\left(\int_0^1 v(p_t)\delta(p_t)dp_t + \int_0^1 v(p_{t+1})\delta(p_{t+1})dp_{t+1}\right) \tag{3}$$

or equivalently

$$\hat{W}(G^{(t,t+1)}) = \frac{1}{T}\sum_{t=1}^{T}\int_0^1 v(p_t)\delta(p_t)dp_t,\ T = 2 \tag{4}$$

Thus, a social evaluation of growth is obtained as the average of the initial- and final-period sensitive growth. The first component of eq. (3), $\int_0^1 v(p_t)\delta(p_t)dp_t$, is a weighted sum of the income change experienced by the individuals that are identified according to their status in the initial period; the second component, $\int_0^1 v(p_{t+1})\delta(p_{t+1})dp_{t+1}$, is a weighted sum of the income change experienced by the individuals that are identified on the base of their status in the final period. The function $v(p_t) : [0,1] \longrightarrow \Re_+,\ t \in \{1,2\}$ expresses the social weight attached to the income change of each individual; this weight depends on the individual's status in the society, as determined by his/her position in the initial ($v(p_t)$) and final ($v(p_{t+1})$) distribution.

Note that $\delta(p_t)$ can be expressed through a variety of measures of individual income growth, including the absolute income change or the proportionate income change.[3]

That is, in order to evaluate growth, one needs to aggregate the income change experienced by each individual, using rank-specific weighting functions. Different preferences over growth processes can be expressible through our model imposing different restrictions on the social weights, hence selecting different classes of weight profiles. These, in turn, define different classes of social evaluation functions (SEF).

The first restriction we impose reflects a standard monotonicity assumption.

Property 1 (*Pro-growth*). $v(p_t) \geq 0$ for all $p_t \in [0,1]$ and for all $t = 1,2$.

It implies that, all else equal, a positive income growth will not decrease social welfare, whereas a negative growth will not increase social welfare.

The second property we consider makes our social evaluation function distribution-sensitive.

Property 2 (*Pro-poor growth*). $\frac{\delta v(p_t)}{\delta p_t} \leq 0$ for all $p_t \in [0,1]$ and for all $t = 1,2$

This property is expression of a transfer-sensitivity principle in the context of income growth among individuals having different ranks in the reference distribution. According to Property 2, decreasing by a given amount the income change of an initially (finally) richer individual and

[2]See also Donaldson and Weymark (1980), Aaberge (2001), and Peragine (2002) for alternative applications.

[3]See Cowell (1985), Fileds and Ok (1999a, 1999b), Schluter and Van de gaer (2011), Palmisano and Van de gaer (2013) for alternative measures of individual income growth that could be used in this work.

increasing by the same amount the income change of an initially (finally) poorer individual will not decrease $\hat{W}$. An income reduction decreases more the social evaluation of growth the poorer is the individual in the initial (final) distribution. In the same vain, an income increase brings more additional welfare the poorer in the initial (final) distribution is the individual experiencing that increase.

Note that Property 1 and 2 capture the main core of our paper as they endorse an agnostic view with respect to the choice of the reference period. In fact, in previous contributions they have been imposed only with respect to $v(p_t)$ for $t = 1$ while letting implicitly $v(p_{t+1}) = 0$. Here, instead, we require that they also hold for $t = 2$.

The next two properties allow for situations in which a social planner would either prefer the initial period status to the final one or the other way round.

Property 3 (*Initial-period relevance*). $v(p_t) \geq v(p_{t+1})$ for all $p_t, p_{t+1} \in [0, 1]$.

Property 3 reflects the idea that the status of individuals in the first period matters more than in the second period. In other words, a social planner would give more relevance to the growth of poor (rich) individuals in the initial period than to the growth of who is poor (rich) in the final period. The following property reflects the opposite argument.

Property 4 (*Final-period relevance*). $v(p_t) \leq v(p_{t+1})$ for all $p_t, p_{t+1} \in [0, 1]$.

According to Property 4, the social evaluation of growth would be more sensitive to the growth experienced by those poor (rich) individuals in the final period than those who are poor (rich) in the initial period. Given that we are not imposing strict inequality, both Property 3 and 4 encompass the special case in which the social evaluation of growth is equally sensitive to the individual's status in the initial and final periods. That is, the growth of the poor (rich) individuals in the initial period affects the social evaluation of growth in the same measure as the growth experienced by the poor (rich) individuals in the final period.

The following families of social evaluation functions can be identified on the base on the properties introduced above:

- $\hat{\mathbf{W}}_1$ is the class of SEFs constructed as in (4) and with social weight functions satisfying Properties 1.
- $\hat{\mathbf{W}}_{1,2}$ is the class of SEFs constructed as in (4) and with social weight functions satisfying Properties 1 and 2.
- $\hat{\mathbf{W}}_{1,3}$ is the class of SEFs constructed as in (4) and with social weight functions satisfying Properties 1 and 3.

- $\hat{\mathbf{W}}_{1,4}$ is the class of SEFs constructed as in (4) and with social weight functions satisfying Properties 1 and 4.

2.3 Results

We now turn to identify a range of conditions to be satisfied for ensuring the dominance of one growth process over the other in terms of extended non-anonymous evaluation, for the different families of social evaluation functions $\hat{\mathbf{W}}$ listed above. All the proofs are gathered in the Theoretical Appendix.

We start considering the class of social evaluation functions $\hat{\mathbf{W}}_1$, for which the following result holds.

Proposition 1 *Given two alternative growth processes, $G_A^{(t,t+1)}$ and $G_B^{(t,t+1)}$, $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_1$, if and only if*

$$(i)\ \delta_A(p_t) \geq \delta_B(p_t)\ \forall p_t \in [0,1] \tag{5}$$

and

$$(ii)\ \delta_A(p_{t+1}) \geq \delta_B(p_{t+1})\ \forall p_{t+1} \in [0,1] \tag{6}$$

Proposition 1 characterizes two dominance conditions of the first order. The first condition requires that the distribution of individuals' income change of growth process A, must lie nowhere below that of B, for all the initial social statuses (or initial income ranks). The second condition requires that the distribution of individuals' income change of growth process A, must lie nowhere below that of B, for all the final social statuses (or final income ranks). Hence, when we only impose pro-growth, to determine which growth process is socially preferable we need to check that for each rank of the initial and final period the growth experienced is higher in A than in B.

Proposition 1 encompasses some interesting special cases. They are summarized in the following Corollary 1 and 2.

Corollary 1 *Given two alternative growth processes, $G_A^{(t,t+1)}$ and $G_B^{(t,t+1)}$, $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_1$, such that $v(p_{t+1}) = 0$, if and only if*

$$\delta_A(p_t) \geq \delta_B(p_t)\ \forall p_t \in [0,1] \tag{7}$$

Corollary 2 *Given two alternative growth processes, $G_A^{(t,t+1)}$ and $G_B^{(t,t+1)}$, $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_1$, such that $v(p_t) = 0$ if and only if*

$$\delta_A(p_{t+1}) \geq \delta_B(p_{t+1})\ \forall p_{t+1} \in [0,1] \tag{8}$$

According to Corollay 1, when we assume that the status of individuals in the final period is not relevant for growth evaluations, Proposition 1 boils down to the standard first order non-anonymous growth dominance condition (see Palmisano and Peragine, 2015; Van Kerm, 2009). According to Corollary 2, instead, we would compare growth processes on the base on growth dominance criteria that are non-anonymous with respect to the identity of individuals only in the final period.

This class of social evaluation functions is the expression of a simple efficiency-based criterion; no concern is expressed in terms of redistributional effects of growth. The next Proposition deals with this issue.

Proposition 2 *Given two alternative growth processes,* $G_A^{(t,t+1)}$ *and* $G_B^{(t,t+1)}$*,* $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1))}, \forall \hat{W} \in \mathbf{W}_{1,2}$*, if and only if*

$$(i) \int_0^{p_t} \delta_A(q_t) dq_t \geq \int_0^{p_t} \delta_B(q_t) dq_t \; \forall p_t \in [0,1] \tag{9}$$

and

$$(ii) \int_0^{p_{t+1}} \delta_A(q_{t+1}) dq_{t+1} \geq \int_0^{p_{t+1}} \delta_B(q_{t+1}) dq_{t+1} \; \forall p_{t+1} \in [0,1] \tag{10}$$

Two conditions of the second order are characterized by this Proposition. According to the first condition, we have to order increasingly individuals on the base of their rank in the initial distribution and check that the cumulated sum of their income change be higher in A than in B. According to the second condition, we have to order increasingly individuals on the base of their rank in the final distribution and check that the cumulated sum of their income change be higher in A than in B. If both conditions are satisfied, under the dominating process initially poor individuals gain more (or lose less) than initially rich and finally poor individuals gain more (or lose less) than finally rich.

As expected, also Proposition 2 encompasses some special cases that are worth observing; they are presented in Corollary 3 and 4.

Corollary 3 *Given two alternative growth processes,* $G_A^{(t,t+1)}$ *and* $G_B^{(t,t+1)}$*,* $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_1$*, such that* $v'(p_{t+1}) = 0$*, if and only if*

$$\int_0^{p_t} \delta_A(q_t) dq_t \geq \int_0^{p_t} \delta_B(q_t) dq_t \; \forall p_t \; \in [0,1] \tag{11}$$

Corollary 4 *Given two alternative growth processes,* $G_A^{(t,t+1)}$ *and* $G_B^{(t,t+1)}$*,* $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_1$*, such that* $v'(p_t) = 0$*, if and only if*

$$\int_0^{p_{t+1}} \delta_A(q_{t+1}) dq_{t+1} \geq \int_0^{p_{t+1}} \delta_B(q_{t+1}) dq_{t+1} \; \forall \; p_{t+1} \in [0,1] \tag{12}$$

Corollary 3 states that when the status of individuals in the final period is not relevant for growth evaluations, the result of Proposition 2 ends up to be equivalent to the standard cumulated non-anonymous growth dominance. In Corollary 4, the status of individuals in the first period is not relevant and to evaluate growth we would only need to check the dominance of the cumulated distribution of the individuals' growth, where these individuals are ordered increasingly on the base of the final income distribution.

Proposition 1 and 2 are agnostic with respect to the choice of the reference period. The next Propositions directly deal with this issue.

Proposition 3 *Given two growth processes* $G_A^{(t,t+1)}$ *and* $G_B^{(t,t+1)}$, $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_{1,3}$, *if and only if*

$$(i)\ \delta_A(p_t)dp_t \geq \delta_B(p_t)dp_t \ \forall \ p_t \in [0,1] \tag{13}$$

and

$$(ii)\ \sum_{t=1}^{T} \delta_A(p_t)dp_t \geq \sum_{t=1}^{T} \delta_B(p_t)dp_t \ \forall \ p_t \in [0,1] \tag{14}$$

Proposition 3 characterizes a sequential dominance condition of the first order. In fact, *(i)* is the first step of this sequential dominance and requires that growth be higher in A than in B at all ranks of the initial distribution. *(ii)* is the second and, in this specific case, the last step of this sequential dominance, given that $t \in \{1,2\}$. It requires that to the growth of the individual ranked p in the initial period we sum the growth of individuals ranked p in the final period. After performing this aggregation for each $p \in [0,1]$ of the initial and final period, we have to check that growth is higher in A than in B, for each aggregated rank-specific growth.

Proposition 4 *Given two growth processes* $G_A^{(t,t+1)}$ *and* $G_B^{(t,t+1)}$, $\hat{W}(G_A^{(t,t+1)}) \geq \hat{W}(G_B^{(t,t+1)}), \forall \hat{W} \in \hat{\mathbf{W}}_{1,4}$, *if and only if*

$$(i)\ \delta_A(p_{t+1})dp_{t+1} \geq \delta_B(p_{t+1})dp_{t+1} \ \forall \ p_{t+1} \in [0,1] \tag{15}$$

and

$$(ii)\ \sum_{t=1}^{T} \delta_A(p_t)dp_t \geq \sum_{t=1}^{T} \delta_B(p_t)dp_t \ \forall \ p_t \in [0,1] \tag{16}$$

Proposition 4 characterizes a 'downward' sequential dominance condition of the first order. *(i)* is the first step of this sequential dominance and requires that growth be higher in A than in B at all ranks of the final distribution. *(ii)* is the second and last step of this sequential dominance. It requires that to the growth of the individual ranked p in the final period we sum the growth of individuals ranked p in the initial period. After performing this aggregation for each $p \in [0,1]$ of the initial and final period, we have to check that growth is higher in A than in B, for each aggregated rank-specific growth. It is clear the difference between Proposition 3 and 4. According to the former, the dominance of A over B is checked starting from a distribution of income change in which individuals are ordered on the base of the first period rank, whereas, according to the latter, we have to start from a distribution of income change in which individuals are ordered on the base of the final period rank.

3 Empirical application

In this Section we implement our theoretical framework in order to analyze the growth process experienced by five different countries in the last decade.[4]

3.1 Data

Our empirical illustration is based on the panel component of the last seven waves of the Cross National Equivalent File (CNEF). The CNEF was designed at Cornell University to provide harmonized data for a set of eight country-specific surveys representative of the respective resident population: the British Household Panel Study (BHPS), the Household Income and Labour Dynamics in Australia (HILDA), the Korea Labor and Income Panel Study (KLIPS), the Swiss Household Panel (SHP), the Canadian Survey of Labour and Income Dynamics (SLID), and the German Socio-Economic Panel (SOEP). In the present paper, we consider Australia, Germany, Korea, Switzerland, and US. In particular, we consider the 2001, 2002, 2009, and 2010 waves for Australia, Germany, and Switzerland, the 1999, 2000, 2007, and 2008 (the last wave available) for Korea, and 1999, 2001, 2007, and 2009 (the last wave available) for US.

The unit of observation is the individual. Our data cover all individuals older than 15. Indviduals with zero sampling weights are excluded since our measures are calculated using sample weights designed to make the samples nationally representative. The measure of living standards is disposable household income, which includes income after transfers and the deduction of income tax and social security contributions. Incomes are expressed in constant 2005 prices, using country and year-specific price indexes and are adjusted for differences in household size, using the square root of the household size. They are then expressed in 2005 Purchasing Power Parity. In line with the literature, for each wave, we drop the bottom and top 1% in the income distribution from the sample to eliminate the effect of possible outliers.

To mitigate the effect of measurement error and transitory income fluctuations, we construct two-year averages of household income for each two-year time period. This implies that we compare the income in 2001-2002 against income in 2009-2010 for Australia, Germany and Switzerland, the income in 1999-2000 against the income in 2007-2008 for Korea, while given that the PSID is conducted every two-years, we compare the income in 1999-2001 against the income in 2007-2009

[4]Note that the content of this empirical illustration is purely descriptive, as the main aim of this section is to show how our framework can be applied on real data.

for US. In order to identify individuals we partition the initial and final distributions of income into 50 quantiles.

We use sample weights to compute all estimates with standard errors obtained through 500 bootstrap replications.[5]

3.2 Results

We now apply the dominance tests presented in Section 2. The results are obtained through pairwise comparisons of the countries analyzed. We start from Proposition 1, where only the size of growth and its direction (positive vs. negative growth) matter.[6]

Table 1 shows that, although the conditions imposed in this proposition are quite strong - it requires a first order dominance of the income change experienced by each individual, where individuals are independently ordered on the base of the first period position in the income ladder and of the second period position - some of the processes can already be ranked. In particular, when relative growth matters, the growth process that took place in Korea dominates those that took place in US, Germany, and Switzerland. The last two processes are also dominated by that of Australia.

Figure 1 plots the na-GIC for each country, where anonymity is expressed with respect to initial period status (panel on the left) and to final period status (panel on the right), corresponding respectively to condition (i) and (ii) of Proposition 1. It is, then, possible to observe that the impossibility of obtaining a solution for the remaining pairwise comparisons is due to the crossing of the na-GICs of the countries considered for some of the initial or final quaintiles. For instance, when initial status matters, Korea' s growth process dominates that of Australia for all the distribution with the exception of the very poorest individuals; while condition (i) requires dominance for all the initial quantiles. A specular situation arises in the comparison concerning Australia and US. Here, Australia's growth dominates that of US for all the distribution with the exception of the richest individuals. We bump into a similar inconclusiveness when we assume that, in this evaluation, individuals are identified on the base of their position in the final distribution.

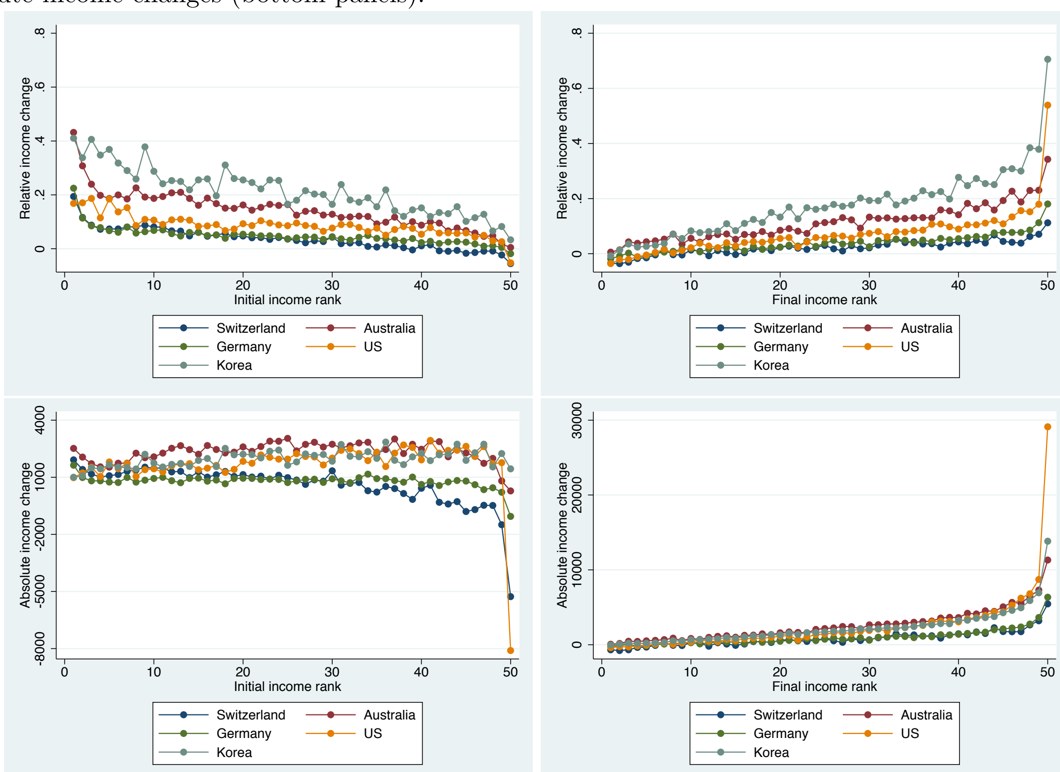
When absolute growth is concerned, inconclusiveness increases and we are only able to prove that the Australian growth episode dominates the German and the Swiss one. The remaining comparisons produce ambiguous results.

We now consider the test proposed in Proposition 2, endorsing the view that priority should be given to the growth experienced by those individuals initially/finally ranked lowest as compared to the growth experienced by those initially/finally ranked highest. Now, when the focus is on relative growth, imposing more restrictions on the social weight helps to increase our ability to

[5]See the Empirical Appendix for more details.

[6]For the sake of brevity, the detailed results for condition (i) and (ii) are reported in the Empirical Appendix.

Figure 1: Non-anonymous growth incidence curves: relative income changes (top panels) and absolute income changes (bottom panels).



Note: anonymity is expressed with respect to initial period status (panels on the left) and to final period status (panels on the right). Source: author' elaboration based on CNEF.

Table 1: Proposition 1

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

rank countries in one more case. This is Australia, whose growth process now dominates that of US.

Figure 2, plotting the cumulated version of the na-GICs presented in Figure 1, shows that, when initial status matters, the ambiguity in the ranking of countries for the remaining comparisons is due to the crossing of the cumulated curve of all countries (with the exception of Australia) in the lowest part of the distribution. When final status matters, instead, intersections of the curves appear only between US and Korea at the very top quantile, and US and Germany at the fifth lowest quantile.

When the focus is on absolute growth, inconclusiveness reduces more. We are able to obtain a ranking in two more cases and they all concern Australia: it ranks the best in all the comparisons executed. Thus, when both the size and the redistributive effect of growth matters, Australia turns out to be the best performing country in terms of (initial and final) non-anonymous growth; whereas the other comparisons produce incomplete results.

We finally perform the tests proposed in Proposition 3 and 4. They account for the possibility that the social planner would either prefer the initial period status to the final one (Proposition 3) or the other way round (Proposition 4).

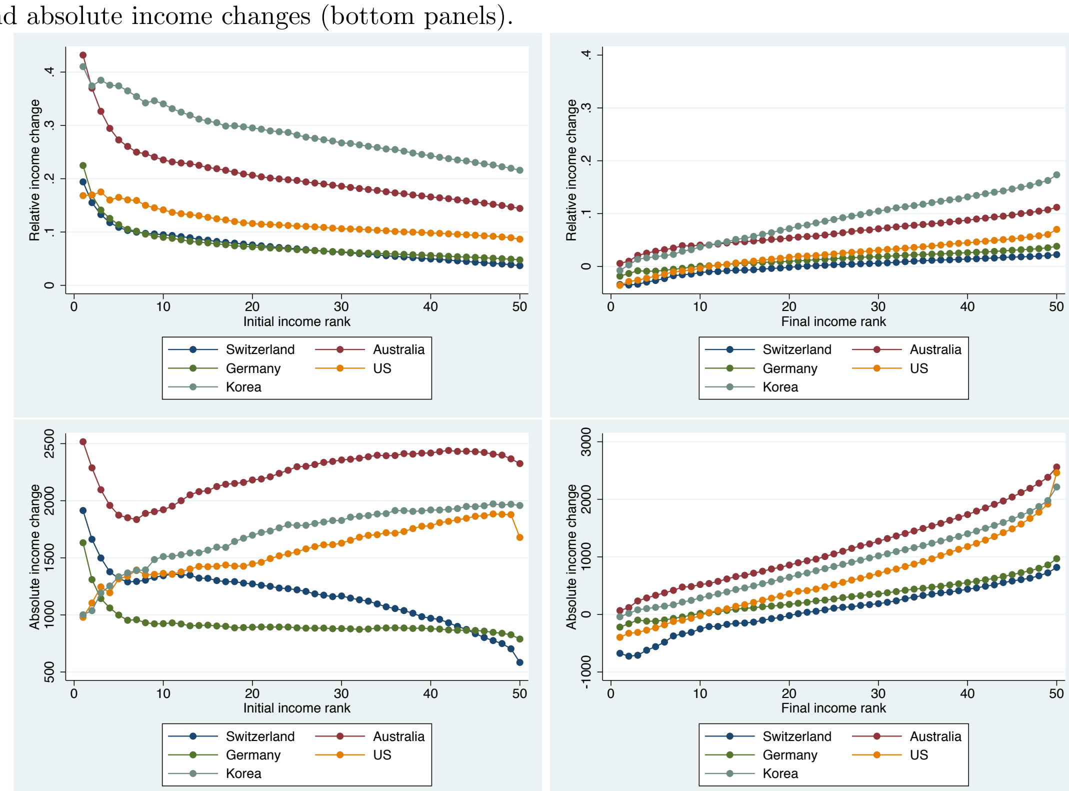
The results of Proposition 3 are reported in Table 3, while those of Proposition 4 are reported in Table 4. Proposition 3 confirms the results found testing Proposition 1. That is, when growth is measured in relative terms the growth taking place in Australia dominates the growth taking place in Germany and Switzerland, while the growth of Korea dominates the growth of Germany, Switzerland and US. As for absolute growth, the only dominance found is between Australia and

Germany and Australia and Switzerland. Proposition 4, instead, confirms the results of Proposition 1 only when we deal with relative growth. When we look at absolute growth we obtain, in addition to the dominance find in Proposition 1, the dominance of Korea over Germany and Switzerland.

Overall it is possible to state that, among the five countries considered, Australia - followed by Korea - arises to be the best performing country, while Germany and Switzerland are the worst performers, both when size and distributional aspects matter in the growth judgment procedure.

Last, it is important to notice that it does make a difference in the ranking of countries whether one is concerned with the initial status of individuals or with their status in the final distribution. Note, in fact, that when the focus is on relative growth, conditions (i) and (ii) in Proposition 2 provide a different result for the ranking of countries (see Table 7 and 8 in the Empirical Appendix). In particular, when first-period status matters (condition (i)), it comes out that we cannot establish a clear ranking between Germany and Switzerland. Whereas, when second-period status matters

Figure 2: Cumulated non-anonymous growth incidence curves: relative income changes (top panels) and absolute income changes (bottom panels).



Note: anonymity is expressed with respect to initial period status (panels on the left) and to final period status (panels on the right). Source: author' elaboration based on CNEF.

Table 2: Proposition 2

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\geq^*$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\geq^*$	$\geq^*$	$\geq^*$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 3: Proposition 3

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 4: Proposition 4

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

(condition (ii)) we obtain that Germany clearly dominates Switzerland. When the focus is on absolute growth, conditions (i) and (ii) provide a different result in both Proposition 1 and Proposition 2. In particular, according to Proposition 1, when second-period status matters (condition (ii)), Germany and Switzerland are dominated by Korea and US, whereas when first-period status matters (condition (i)) no clear ranking can be established between these countries. As for Proposition 2, it comes out that Germany is dominated by Korea and Switzerland is dominated by Korea, Germany and US according to condition (ii), while these rankings do not hold anymore according to condition (i). Most importantly, the different conclusions generated by Proposition 3 and 4, as concerned absolute growth, are insightful (see Table 9, 10, 11, and 12 in the Empirical Appendix). It can be noticed that the dominance of Korea over Germany and Switzerland that is found in Proposition 4 is not confirmed in Proposition 3, although such dominance arises in condition (ii) of the latter. This means that the extent of the difference in the evaluation of growth between Korea and Germany and Korea and Switzerland, when final-period status matters, is such that it is able to compensate for the absence of difference in the evaluation of growth between these three countries that is found when initial-period status matters. Moreover, when more relevance is put on the second-period as the reference one to identify individuals, it is possible to rank the countries in two more cases (Korea over Germany and Switzerland) with respect to a situation in which more relevance is given to the first-period as the reference distribution.

We conclude our analysis by performing some robustness check related to variation in household composition. It might be argued that the results of our analysis are sensitive to changes in household composition between the initial (first) period and the second period of the growth process. Hence we recalculated our estimates using, for each growth process only the subsample of households which did not change in composition between the initial and second period. The results, reported in the empirical appendix, show that our conclusions are not affected.

4 Conclusions

An increasing number of contributions in recent years has proposed alternative models to evaluate and rank growth processes that account for the identity of individuals, where the identity has been represented by their relative position in the pre-growth distribution of income. In this work we have generalized this non-anonymous approach by providing a normative framework to rank growth processes that is robust to the choice of the reference period used to identify individuals.

In particular, we have adopted a bi-dimensional framework, where the two dimensions are respectively the rank of the individual in the income distribution of the reference period and the income change experienced by each individual. We have, then, provided partial dominance conditions for ordering growth processes and we have shown how they relate to the existing conditions in the literature.

We have used this framework to assess and rank the growth processes that took place in the last decade in five different countries: Australia, Germany, Korea, Switzerland, and US. Our results show that Australia, followed by Korea, arises to be the best performing country, while Germany and Switzerland arise to be the worst performing countries, when both initial and final period are relevant reference periods to identify individuals.

The results derived in our paper can be extended in a number of directions. First, new dominance conditions can be obtained if both property 2 and 3 (or property 2 and 4) are imposed on the same social evaluation function, such that it is possible to account for both 'progressive' concerns and time relevance concerns. The resulting dominance conditions would help to increase the possibility of ordering countries, although with the cost of further restricting the family of social evaluation functions to which such conditions would apply. Second, the framework proposed in this paper could be extended to endorse an intertemporal perspective, as recently explored in Bresson et al. (2015), that does not simply compare in a non-anonymous fashion the initial and the final period but is able to account for the income and status variation of individuals between these two periods. These extensions will be the object of future research.

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Theoretical Appendix

Proof of Proposition 1

We want to find sufficient and necessary conditions such that

$$\Delta\hat{W} = \frac{1}{T}\sum_{t=1}^{T}\int_0^1 v(p_t)(\delta_A(p_t)dp_t - \delta_B(p_t)dp_t) \geq 0, for\ all\ \hat{W} \in \hat{\mathbf{W}}_1,\ T=2 \tag{17}$$

Letting $\Delta\delta(p_t)dp_t = \delta_A(p_t)dp_t - \delta_B(p_t)dp_t$ and $\Delta\delta(p_{t+1})dp_{t+1} = \delta_A(p_{t+1})dp_{t+1} - \delta_B(p_{t+1})dp_{t+1}$, rewrite eq.(17) as follows:

$$\Delta\hat{W} = \frac{1}{T}(\int_0^1 v(p_t)\Delta\delta(p_t)dp_t + \int_0^1 v(p_{t+1})\Delta\delta(p_{t+1})dp_{t+1}) \geq 0 \tag{18}$$

For the sufficiency part, by Property 1 $v(p_t), v(p_{t+1}) \geq 0$ for all $p_t, p_{t+1} \in [0,1]$, then $\Delta\delta(p_t)$ for all $p_t \in [0,1]$ and $\Delta\delta(p_{t+1}) \geq 0$ for all $p_{t+1} \in [0,1]$ imply $\int_0^1 v(p_t)\Delta\delta(p_t) \geq 0$ and $\int_0^1 v(p_{t+1})\Delta\delta(p_{t+1}) \geq 0$, which imply $\Delta\hat{W} \geq 0$.

For the necessity part, let $\Delta\hat{W} \geq 0$, but assume that $\Delta\delta(p_t) < 0$ for some $p_t \in [0,1]$ and that $\Delta\delta(p_{t+1}) < 0$ for some $p_{t+1} \in [0,1]$. Following Lemma 1 in Chambaz and Maurin (1998), there exists a set of values $z(p) \in V^+$ and $\rho(p) \in V^+$ such that $\int_0^1 z(p)\Delta\delta(p_t)dp_t \leq 0$ and $\int_0^1 \rho(p)\Delta\delta(p_{t+1})dp_{t+1} \leq 0$. Define $z(p) = v(p_t)$ and $\rho(p) = v(p_{t+1})$, since $z(p), \rho(p) \in V^+$ they satisfy Property 1, now substituting in eq. (18) gives $\Delta\hat{W} \leq 0$ which is a contradiction. **QED**

Proof of Proposition 2

We want to find sufficient and necessary conditions such that

$$\Delta\hat{W} = \frac{1}{T}(\int_0^1 v(p_t)\Delta\delta(p_t)dp_t + \int_0^1 v(p_{t+1})\Delta\delta(p_{t+1})dp_{t+1}) \geq 0, for\ all\ \hat{W} \in \hat{\mathbf{W}}_{1,2},\ T=2 \tag{19}$$

For the sufficiency part, we integrate by parts eq. (19):

$$v(p_t = 1)\int_0^1 \Delta\delta(p_t)dp_t - \int_0^1 v'(p_t)\int_0^{p_t} \Delta\delta(q_t)dq_t + \tag{20}$$

$$+v(p_{t+1} = 1)\int_0^1 \Delta\delta(p_{t+1})dp_{t+1} - \int_0^1 v'(p_{t+1})\int_0^{p_{t+1}} \Delta\delta(q_{t+1})dq_{t+1}$$

Since by Property 1 $v(p_t = 1), v(p_{t+1} = 1) \geq 0$ for all $p_t, p_{t+1} \in [0,1]$, $\int_0^{p_t} \Delta\delta(q_t)dq_t$ for all $p_t \in [0,1]$ and $\int_0^{p_{t+1}} \Delta\delta(q_{t+1})dq_{t+1} \geq 0$ for all $p_{t+1} \in [0,1]$ imply $v(p_t = 1)\int_0^1 \Delta\delta(p_t)dp_t \geq 0$ and $v(p_{t+1} = 1)\int_0^1 \Delta\delta(p_{t+1})dp_{t+1} \geq 0$. Furthermore, by Property 2 $v'(p_t) \leq 0$ and $v'(p_{t+1}) \leq 0$ for all $p_t, p_{t+1} \in [0,1]$, we have $\int_0^1 v'(p_t)\int_0^{p_t} \Delta\delta(q_t)dq_t \leq 0$ and $\int_0^1 v'(p_{t+1})\int_0^{p_{t+1}} \Delta\delta(q_{t+1})dq_{t+1} \leq 0$. Thus, $\Delta\hat{W} \geq 0$.

For the necessity part, let $\Delta\hat{W} \geq 0$, but assume that $\int_0^{p_t} \Delta\delta(q_t)dq_t < 0$ for some $p_t \in [0,1]$ and that $\int_0^{p_{t+1}} \Delta\delta(p_{t+1})dp_{t+1} < 0$ for some $p_{t+1} \in [0,1]$. Rewrite eq.(20) as follows:

$$v(p_t = 1)\int_0^1 \Delta\delta(p_t)dp_t + \int_0^1 -v_t'(p_t)\int_0^{p_t} \Delta\delta(q_t)dq_t + \tag{21}$$

$$+v_1(1)\int_0^1 \Delta\delta(p_2)dp_2 + \int_0^1 -v_2'(p_2)\int_0^{p_2} \Delta\delta(q_2)dq_2$$

Denote $-v'(p_t) = \alpha(p)$ and $-v'(p_{t+1}) = \beta(p)$. By Lemma 2 in Chambaz and Maurin (1998), $\int_0^1 \alpha(p)\int_0^{p_t} \Delta\delta(q_t)dq_t \leq 0$ for all $\alpha(p) \in V^+$ and $\int_0^1 \beta(p)\int_0^{p_{t+1}} \Delta\delta(q_{t+1})dq_{t+1} \leq 0$ for all $\beta(p) \in V^+$. Hence, the second and fourth term of eq. (21) must be negative. Then it is always possible to find combinations of $v(p_t)$, $v(p_{t+1})$ and $\Delta\delta(p_t)$, $\Delta\delta(p_{t+1})$ such that

$$\left| v_(p_t = 1)\int_0^1 \Delta\delta(p_t)dp_t + v(p_{t+1} = 1)\int_0^1 \Delta\delta(p_{t+1})dp_{t+1} \right| < \tag{22}$$

$$\left| \int_0^1 -v'(p_t)\int_0^{p_t} \Delta\delta(q_t)dq_t + \int_0^1 -v'(p_{t+1})\int_0^{p_{t+1}} \Delta\delta(q_{t+1})dq_{t+1} \right|$$

which results in $\Delta\hat{W} < 0$, a contradiction. **QED**

Proof of Proposition 3.

We want to find sufficient and necessary conditions such that

$$\Delta\hat{W} = \frac{1}{T}\sum_{t=1}^{T}\int_0^1 v(p_t)\Delta\delta(p_t)dp_t \geq 0,\ for\ all\ \hat{W} \in \hat{\mathbf{W}}_{1,3},\ T = 2 \tag{23}$$

Sufficiency can be shown as follows. First, reverse the order of integration and summation, such that

$$\Delta\hat{W} = \frac{1}{T}\int_0^1 \sum_{t=1}^{T} v\left(p_t\right)\Delta\delta(p_t)dp_t \geq 0 \tag{24}$$

Since by Property 1 and 4 $v\left(p_t\right) \geq v\left(p_{t+1}\right) \geq 0$ $\forall p_t, p_{t+1} \in [0,1]$, we can apply the Abel's Lemma and obtain that $\sum_{t=1}^{T}\left(p_t\right)\Delta\delta\left(p_t\right) \geq 0$ if $\sum_{t=1}^{k}\Delta\delta\left(p_t\right) \geq 0, \forall k = 1,...,T$ and $\forall p_t \in [0,1]$. It follows that $\sum_{t=1}^{T} v\left(p_t\right)\Delta\delta\left(p_t\right) \geq 0$, $\forall p_t \in [0,1]$, implies that, integrating with respect to p_t, $\int_0^1 \sum_{t=1}^{T} v\left(p_t\right)\Delta\delta\left(p_t\right)dp_t \geq 0$.

For the necessity, suppose for a contradiction that $\Delta\hat{W} \geq 0$, but there is a period $\tau \in \{1,...,T\}$ and an interval $I \equiv [a,b] \subseteq [0,1]$ such that $\sum_{t=1}^{\tau}\Delta\delta\left(p_t\right) < 0, \forall p_t \in I$. Now, applying Abel's Lemma, there exists a set of functions $\{v\left(p_t\right) \geq 0\} : [0,1] \longrightarrow \Re_+$, $t = 1,...,T$, such that $\sum_{t=1}^{T} v\left(p_t\right)\Delta\delta\left(p_t\right) < 0$, $\forall p_t \in I$. Writing $\sum_{t=1}^{T} v\left(p_t\right)\Delta\delta(p_t) = \Gamma\left(p_t\right)$, $\Delta\hat{W}$ reduces to $\int_0^1 \Gamma\left(p_t\right)dp_t$, where $\Gamma\left(p_t\right) < 0$,

$\forall p_t \in I$. Selecting a set of function $\Gamma(p_t)$, such that $\Gamma(p_t) \longrightarrow 0$, $\forall p_t \in [0,1] \setminus I$, $\Delta\hat{W}$ would reduce to $\int_a^b \Gamma(p_t)\,dp_t < 0$, a contradiction. **QED**

Proof of Proposition 4.

Before proving this proposition we need to state and prove the following lemma.

Lemma 1.

$\sum_{i=1}^{n} v_i w_i \geq 0$ *for all sets of number* $\{v_i\}$ *such that* $0 \leq v_1 ... \leq v_i \leq v_{i+1} \leq ... \leq v_n$ *for all* $i \in \{1,...,n\}$, *if and only if* $\sum_{i=j}^{n} w_i \geq 0$ *for each* j.[7]

Proof For the sufficiency, note that $\sum_{i=1}^{n} v_i w_i$ can be decomposed as $\sum_{i=1}^{n} v_i w_i = v_1 \sum_{i=1}^{n} w_i + \sum_{i=1}^{n-1}(v_{i+1} - v_1)\sum_{j=i}^{n} w_j = v_1 \sum_{i=1}^{n} w_i + (v_2 - v_1)\sum_{i=2}^{n} w_i + (v_3 - v_2)\sum_{i=3}^{n} w_i + ... + (v_{n-2} - v_{n-3})\sum_{i=n-2}^{n} w_i + (v_{n-1} - v_{n-2})\sum_{i=n-1}^{n} w_1 + (v_n - v_{n-1})w_n$. It is clear that $\sum_{i=j}^{n} w_i \geq 0$ for each j for each j implies $\sum_{i=1}^{n} v_i w_i \geq 0$.

As for the necessity part, suppose that $\sum_{i=1}^{n} v_i w_i \geq 0$ for all sets of numbers $\{v_i\}$ such that $0 \leq v_1 ... \leq v_i \leq v_{i+1} \leq ... \leq v_n$, but $\exists j \in 1,...,n$ such that $\sum_{i=j}^{n} w_i < 0$, then consider what happens when $v(1) \searrow 0$ and $v_{i+1} - v_i \searrow 0$ for all $i \neq j$: $\sum_{i=1}^{n} v_i w_i = v_1 \sum_{i=1}^{n} w_i + \sum_{i=1}^{n-1}(v_{i+1} - v_1)\sum_{j=i}^{n} w_j < 0$, which is a contradiction. **QED**

We want to find sufficient and necessary conditions such that

$$\Delta\hat{W} = \frac{1}{T}\sum_{t=1}^{T}\int_0^1 v(p_t)\Delta\delta(p_t)dp_t \geq 0,\ for\ all\ \hat{W} \in \hat{\mathbf{W}}_{1,4},\ T = 2 \tag{25}$$

Sufficiency can be shown as follows. First, reverse the order of integration and summation, such that

$$\Delta\hat{W} = \int_0^1 \sum_{t=1}^{T} v(p_t)\,\Delta\delta(p_t dp_t) \geq 0 \tag{26}$$

Since by Property 1 $v(p_t) \geq 0$ and $v(p_{t+1}) \geq 0$ $\forall p_t, p_{t+1} \in [0,1]$, and by Property 4 $v(p_{t+1}) \geq v(p_t)$, we can apply Lemma 1 and obtain that $\sum_{t=1}^{T}(p_t)\,\Delta\delta(p_t) \geq 0$ if $\sum_{t=k}^{T}\Delta\delta(p_t) \geq 0, \forall k = 1,...,T$ and $\forall p_t \in [0,1]$. It follows that $\sum_{t=1}^{T} v(p_t)\,\Delta\delta(p_t) \geq 0$, $\forall p_t \in [0,1]$, implies that, integrating with respect to p_t, $\int_0^1 \sum_{t=1}^{T} v(p_t)\,\Delta\delta(p_t)\,dp_t \geq 0$.

For the necessity the proof follows as in Proposition 3, suppose for a contradiction that $\Delta\hat{W} \geq 0$, but there is a period $\tau \in \{1,...,T\}$ and an interval $I \equiv [a,b] \subseteq [0,1]$ such that $\sum_{t=\tau}^{T}\Delta\delta(p_t) < 0, \forall p_t \in I$. Now, applying Lemma 1, there exists a set of functions $\{v(p_t) \geq 0\} : [0,1] \longrightarrow \Re_+$, $t = 1,...,T$, such that $\sum_{t=1}^{T} v(p_t)\,\Delta\delta(p_t) < 0$, $\forall p_t \in I$. Writing $\sum_{t=1}^{T} v(p_t)\,\Delta\delta(p_t) = \Gamma(p_t)$, $\Delta\hat{W}$ reduces to $\int_0^1 \Gamma(p_t)\,dp_t$, where $\Gamma(p_t) < 0$, $\forall p_t \in I$. Selecting a set of function $\Gamma(p_t)$, such that $\Gamma(p_t) \longrightarrow 0$, $\forall p_t \in [0,1] \setminus I$, $\Delta\hat{W}$ would reduce to $\int_a^b \Gamma(p_t)\,dp_t < 0$, a contradiction. **QED**

[7] Note that this is different from the Abel's Lemma, which states that a sufficient condition for $\sum_{i=1}^{n} v_i w_i \geq 0$ for all sets of $\{v_i\}$ such that $v_1 ... \geq v_i \geq v_{i+1} \geq ... \geq v_n$ is $\sum_{i=1}^{j} w_i \geq 0$ for each j.

Lemma 2 *For all F and $G \in \Psi$, $W(F) \geq W(G)$ for all $W(F) = \frac{1}{n}(\sum_i^n v_i F_i^{-1})$ and $W = \frac{1}{n}(\sum_i^n v_i G_i^{-1})$ such that $\sum_{i=1}^n v_i = 1$, $v_i \geq 0$ and $0 \leq v_1 \leq ... \leq v_i \leq ... \leq v_{i+1} \leq ... \leq v_n$ if and only if*

$$\sum_{i=k}^{n} F_i^{-1} \geq \sum_{i=k}^{n} G_i^{-1} \text{ for all } k \in \{1, ..., n\} \tag{27}$$

Proof $\Delta W = \frac{1}{n}(\sum_i^n v_i(F_i^{-1} - G_i^{-1})$. Denoting $\Delta_i = (F_i^{-1} - G_i^{-1})$, rewrite $\Delta W = \frac{1}{n}(\sum_i^n v_i \Delta_i)$. Now rewrite $\Delta W = v_1 \sum_i^n \Delta_i + \sum_{i=1}^{n-1}(v_{i+1} - v_i) \sum_{i=j}^n \Delta_i$. Given that $v_i \geq 0$ and $0 \leq v_1 \leq ... \leq v_i \leq ... \leq v_{i+1} \leq ... \leq v_n$ we can apply Lemma 1 to get that $\Delta W \geq 0$ if and only if $\sum_{i=j}^n \Delta_i \geq 0$ for all $j \in \{1, ..., n\}$. **QED**

Empirical Appendix

5 Bootstrap Procedure

To take into account the dependence structure of our observations, we use the non-parametric bootstrap procedure described by Cameron and Trivedi (2010). In this procedure, the bootstrap samples are obtained by implementing the `bsweight` stata routine proposed by Kolenikov (2010), which takes the stratification of data into account.

Let Y^b be the $b-th$ bootstrap replication of the full sample, with $b = 1, ..., B$ and $B = 1000$. Let then $S_t(Y^b)$ be the replication b subsample for period $(t, t{+}2)$, with $t = \{1998, 2000, 2002, 2004, 2008\}$. All our indices and their differences are estimated on each replicate subsample $S_t(Y^b)$ and we denote it by $\hat{\beta}_t^b = \beta\left(S_t(Y^b)\right)$.

The standard error of the statistic $\hat{\beta}_t^b$ is obtained as:

$$\hat{\sigma} = \sqrt{\sum_{b=1}^{B}\left(\hat{\beta}_t^b - \bar{\beta}_t\right)^2 /(B-1)}$$

where $\bar{\beta}_t = \frac{\sum_{b=1}^{B}\hat{\beta}_t^b}{B}$.

The lower and upper confidence bounds are the $B * \alpha/2 - th$ and $B * (1 - \alpha/2) - th$ ordered elements, respectively. For $B = 1000$ and $\alpha = 5\%$ these are the $25th$ and $975th$ ordered elements of the empirical distribution $F\left(\hat{\beta}_t\right)$.

6 Detailed Results

Table 5: Proposition 1, condition (i)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 6: Proposition 1, condition (ii)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\leq^*$
Korea				$\geq^*$	$\oslash$
Switzerland					$\leq^*$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 7: Proposition 2, condition (i)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\geq^*$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\geq^*$	$\geq^*$	$\geq^*$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 8: Proposition 2, condition (ii)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\geq^*$
Germany			$\leq^*$	$\geq^*$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\geq^*$	$\geq^*$	$\geq^*$
Germany			$\leq^*$	$\geq^*$	$\oslash$
Korea				$\geq^*$	$\oslash$
Switzerland					$\leq^*$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 9: Proposition 3, condition (i)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 10: Proposition 3, condition (ii)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 11: Proposition 4, condition (i)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\leq^*$
Korea				$\geq^*$	$\oslash$
Switzerland					$\leq^*$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 12: Proposition 4, condition (ii)

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

7 Controlling for changes in household size

Table 13: Proposition 1

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\oslash$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

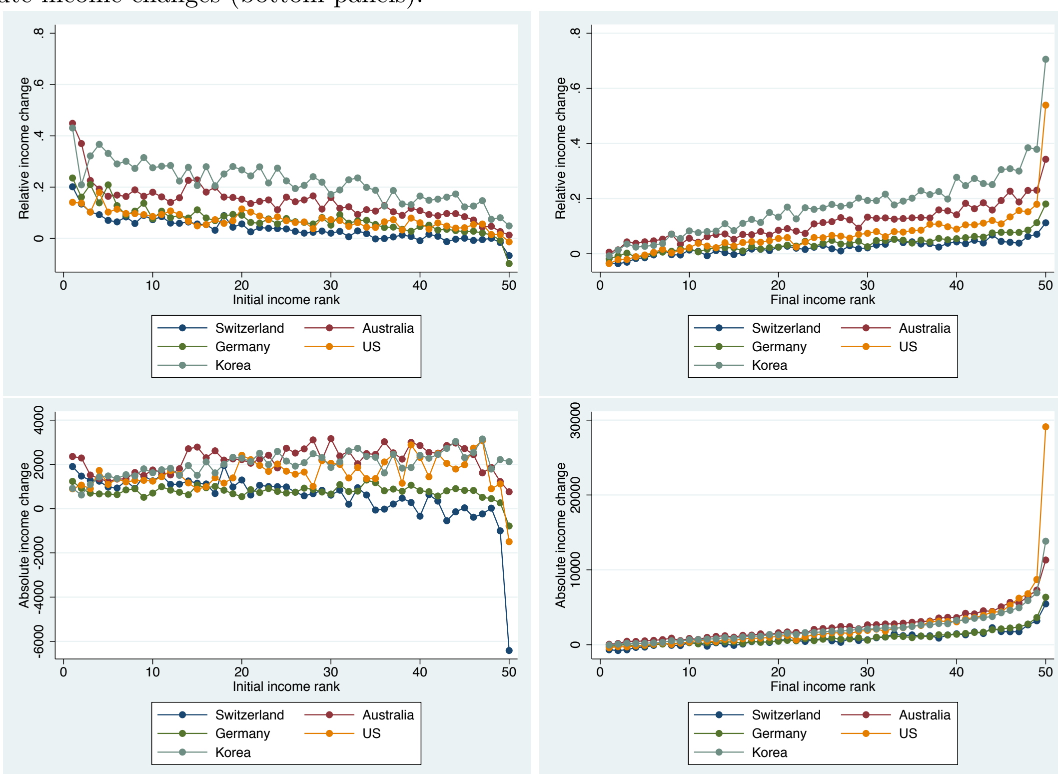
Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 14: Proposition 2

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\geq^*$
Germany			$\leq^*$	$\geq^*$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\geq^*$	$\geq^*$	$\geq^*$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

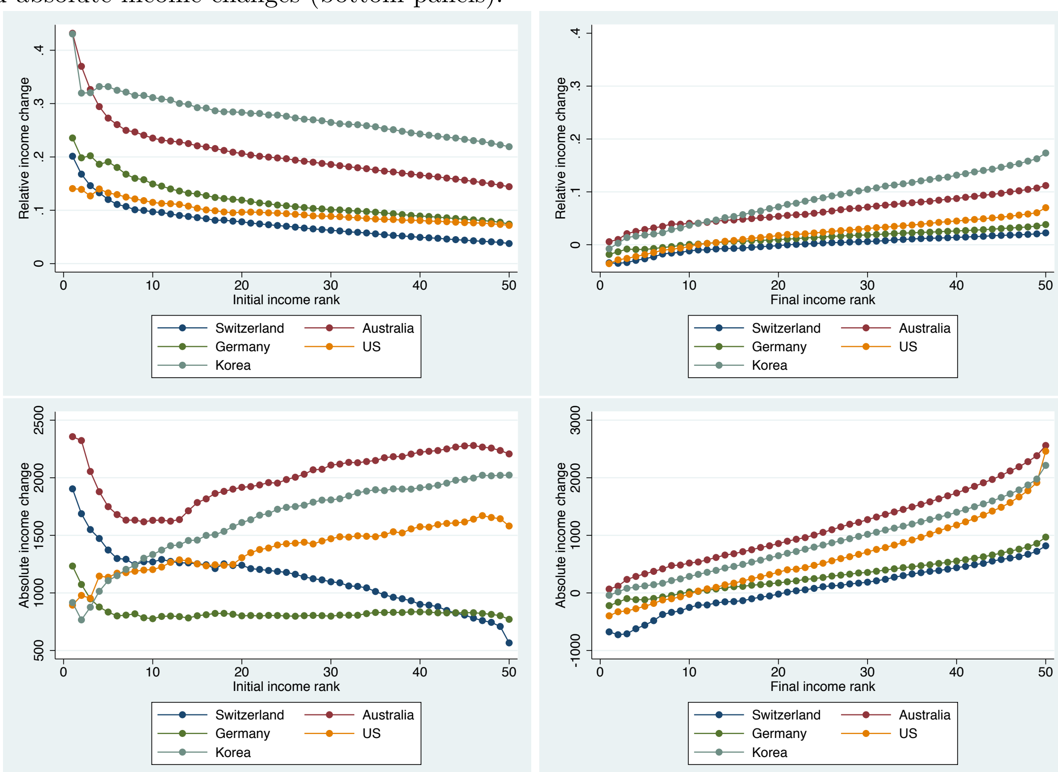
Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Figure 3: Non-anonymous growth incidence curves: relative income changes (top panels) and absolute income changes (bottom panels).



Note: anonymity is expressed with respect to initial period status (panels on the left) and to final period status (panels on the right). Source: author' elaboration based on CNEF.

Figure 4: Cumulated non-anonymous growth incidence curves: relative income changes (top panels) and absolute income changes (bottom panels).



Note: anonymity is expressed with respect to initial period status (panels on the left) and to final period status (panels on the right). Source: author' elaboration based on CNEF.

Table 15: Proposition 3

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\oslash$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.

Table 16: Proposition 4

	Relative income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\leq^*$	$\oslash$	$\oslash$
Korea				$\geq^*$	$\geq^*$
Switzerland					$\oslash$
	Absolute income change				
	Australia	Germany	Korea	Switzerland	US
Australia		$\geq^*$	$\oslash$	$\geq^*$	$\oslash$
Germany			$\oslash$	$\oslash$	$\oslash$
Korea				$\oslash$	$\oslash$
Switzerland					$\oslash$

Note: $\geq$ ($\leq$) indicates that the first distribution dominates (is dominated by) the second distribution. $\oslash$ denotes a non-conclusive test. '$*$' means significant at 95 %. Source: author's elaboration based on CNEF.