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e metodi matematici

From fixed to state-dependent duration in public-private partnerships

Daniel Danau & Annalisa Vinella

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Daniel Danau†   Annalisa Vinella‡

Abstract

A government delegates a build-operate-transfer project to a private firm. In the contracting stage, the operating cost is unknown. The firm can increase the likelihood of facing a low cost, rather than a high cost, by exerting costly effort when building the infrastructure. Once this is in place, the firm learns the true cost and begins to operate. We show that, under limited commitment, if the break-up of the partnership is sufficiently costly to the government and/or information problems are sufficiently severe, the contract is not robust to renegotiation unless it has a longer duration when the realized cost is low. This result is at odds with the prescription of the literature on flexible-term contracts, which recommends a longer duration when operating conditions are unfavourable.

Keywords: Public-private partnerships; state-dependent duration; flexible-term contract; limited commitment; renegotiation; break-up

J.E.L. Classification Numbers: D82; H57; H81

1 Introduction

Public-private partnerships (PPPs) in infrastructure projects include two main phases, namely construction and operation, and it is well known that incentive problems affect their performance in either phase. When building the infrastructure, the private firm may be little motivated to exert costly effort (Hart [13], Bennett and Iossa [2], Martimort and Pouyet [19], Iossa and Martimort [16]). In the operation phase, the firm is likely to observe the operating conditions privately, as the agency theory suggests, and to camouflage them *vis-à-vis* the government (e.g. Laffont [17], Guasch *et al.* [10] - [11], Iossa and Martimort [16], Danau and Vinella [3]). Moreover, both the firm and the government may have an interest to abjure the PPP contract (see the report of Guasch [9] and the cases described by Estache and Wren-Lewis [8]). The length of the contract plays an essential role in addressing these incentive problems (Danau and Vinella [3]). This is because variations in the contractual length grant to the contract designer some flexibility in adjusting the per-period compensation, which can be exploited to solve incentive problems in operation, without affecting the total compensation, which is used to address moral hazard in construction instead. The available flexibility depends on the severity of moral hazard. The theory of incentives tells us that the more severe is moral hazard, the more uncertain the total compensation should be. However, as the firm is exposed to more risk, there is less flexibility in adjusting the per-period compensation through changes in the contractual length. Hence, it becomes more difficult to incentivize the firm in operation. The choice of a suitable contractual length is thus related to how important each of the incentive problems is.

In spite of this essential link between the duration of PPP contracts and the partners' incentives in construction and operation, the choice of the optimal contractual length in PPP projects is still under-explored. Particularly, to the best of our knowledge, the literature on agency relationships has not yet considered the possibility of conditioning the duration of the contract on the state of nature as a tool to solve incentive problems. The idea of a state-dependent duration is inspired to the studies of Engel *et al.* [6] - [7]. They focus on frameworks where fixed-term contracts are incomplete, and show that incompleteness is eliminated if the contractual length is adjusted according to the realized state of nature in such a way that the firm attains its reservation utility regardless of the specific state. This requires letting the firm run the activity for a bigger number of periods when the operating conditions are unfavourable. Contracts with this characteristic are referred to as flexible-term contracts. However, one may wonder whether the benefits of the flexible-term contracts extend also to the frameworks we have in mind, where information problems have bite and contractual frictions are due to the lack of enforcement mechanisms rather than to contractual incompleteness (as in the cases

described by Estache and Wren-Lewis [8]).[1] As we said, information problems require that the firm be exposed to some risk. This involves setting the compensation to the firm lower in bad states of nature. Besides, if in bad states the contract has a long duration, as is the case of flexible-term contracts, then, as time goes by, the firm might prefer to cease honouring its obligations, provided that its residual compensation falls below some alternative opportunity, which could be derived from another activity or from a new deal with the same partner. One then needs to understand whether a contract with a state-dependent duration would be useful in the environments we consider and, if so, how it should exactly be structured. We explore these issues in our paper.

The analysis we develop delivers one main lesson. In situations where the firm enjoys an informational advantage early on in the relationship with the government and, in addition, it may behave opportunistically during the operation phase, the contract that stipulates an efficient allocation cannot be made renegotiation-proof, unless the contractual length is conditioned on the state of nature. Furthermore, at odds with the flexible-term policy, the contractual length should be set longer in favourable states than in unfavourable states. A contract with this characteristic is more likely to be necessary the more severe are moral hazard and adverse selection and/or the higher is the cost that the break-up of the partnership would occasion to the government.[2]

This study is related to Danau and Vinella [3], based on which the model is built. However, while that paper explores the financial structure of the project, here we assume that the firm is the only investor. This simplification does not affect the general insights of our study. Yet it is functional to making the analysis well focused on the use of a state-dependent duration as an incentive tool in PPP contracts. At the same time, it leads to a complication. In any period in which some partner were to breach the contract, renegotiation would be Pareto-improving on the termination of the partnership. Hence, following a contractual breach, the parties would actually reach a new agreement and continue the relationship. The contract must thus be robust to the possibility of repeated renegotiation. In Danau and Vinella [3], the convenience of seeking new deals is ruled out by the presence of a financial institution which can impose high debt payments to the government so as to destroy any surplus to be shared in renegotiation.

As compared to the literature on flexible-term contracts, there are two essential differences

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[1]Laffont [17], Guasch *et al.* [10] - [11], Danau and Vinella [3] are theoretical studies in which the vulnerability of the contract follows from the lack of enforcement mechanisms in the economy. Similarly to Engel *et al.* [6] - [7], Iossa and Martimort [15] rely as well on an incomplete contracting approach to model the vulnerability of PPP contracts.

[2]A good example of the cost that governments fears in the event of a break-up is provided by Ehrhardt and Irwin [5] with regards to the 1999 Melbourne transport franchises. The authors report that, on being asked to renegotiate, the State Government of Victoria preferred to escape the additional expenses, associated with retrieval of the activities and possible litigations, which the break-up of the relationship would occasion. See also Trebicock and Rosenstock [21], who acknowledge that governments face transaction costs when PPPs are broken up.

in our framework, which pave the way for a different result. First, the compensation to the firm is endogenous and it is used as a tool to fine tune incentives. Second, it is necessary to make explicit analytical consideration of the renegotiation game in which the partners engage following a contractual breach. This is not the case, instead, in the framework of Engel *et al.* [6] - [7], where renegotiation could only follow from contractual incompleteness, an issue which is removed by making the firm's payoff independent of the operating conditions.

Beside the studies on PPPs aforementioned, our analysis is more generally related to those on long-term principal-agent relationships. Baron and Besanko [1] characterize the optimal dynamic contract in a repeated adverse-selection regulatory problem. Dewatripont [4], Hart and Tirole [14], Rey and Salanié [20] show that the parties to an incentive contract, signed at *interim*, may want to renegotiate the allocation initially stipulated, once private information is revealed. This desire arises because, under complete information, a Pareto-improving allocation is available to the contractual parties. More recently, awareness has been shown of the importance that limits to the enforcement ability of the courts of justice may have in contractual design (see Levin [18], who focuses on relational contracts). Unlike in this domain of literature, in the PPP context we represent, the contract specifies a single intertemporal compensation to the firm and the choice of the contractual length involves how that compensation is spread over multiple periods. In the same vein as Levin [18], we identify conditions under which the result of Harris and Raviv [12] that efficiency is attained with *ex-ante* contracting does hold even if the contractual parties are unable to commit. The specificity of our analysis is that this result rests on the way in which the duration of the contract is chosen by the principal.

OUTLINE   The reminder of the paper is organized as follows. The model is described and the efficient allocation is characterized in section 2. In section 3, we identify termination dates such that information problems are addressed without inducing distortions away from efficiency in the contractual allocation. In section 4, we show how a state-dependent duration can be useful to make the contract robust to renegotiation. Section 5 investigates the flexibility gain that a state-dependent duration grants to the contract designer. Section 6 briefly concludes.

2   Basic setup and efficient allocation

A government (G) delegates a public project to a private firm (F). The project includes the construction and the management of an infrastructure to be used to provide a good (or service) to society. F is a Special-Purpose-Vehicle (SPV), expressly created by a group of private investors to perform these tasks. The contract is signed and the infrastructure is built at the beginning of period 0. The infrastructure is managed in all periods $\tau \in \{0, ..., T-1\}$. At the beginning of period T the contract ends. For simplicity, we assume that the infrastructure has

an infinite life, during which it does not depreciate. If T is finite, then the infrastructure is transferred to G at the end of the contract, as is typical of PPP arrangements.

At the beginning of period 0 F sinks a cost of $I > 0$ to construct the infrastructure. In each period τ, F runs the production process incurring a cost of θq, where $\theta > 0$ is the unit cost and $q \geq 0$ is the number of units of the good delivered at the end of the period. In return for supply, F receives a transfer of t from G and collects revenues $p(q)q$ from the market. The per-period profit of F is, thus, $\pi = t + (p(q) - \theta) q$. Consumption of q units of the good yields a gross surplus of $S(q)$, such that $S'(\cdot) > 0$, $S''(\cdot) < 0$, $S(0) = 0$, and the Inada's conditions hold. Customers purchase the output produced in each τ at a price of $p(q) \equiv S'(q)$. G attaches to the project a value equal to consumer surplus net of the transfer made to the firm. The per-period value of the project to G is, thus, $S(q) - (t + p(q)q)$.[3]

The unit cost θ is unknown when the contract is signed. Its distribution depends on some unobservable effort $a \in \{0, 1\}$ that F exerts when constructing the infrastructure. Once the infrastructure is in place and F begins to operate, the unit cost is realized, taking one of the two possible values, namely θ_l and θ_h, such that $0 < \theta_l < \theta_h$. Provided θ is an inner characteristic of the infrastructure, its realization remains unchanged during the life of the project. Henceforth, we denote $i \in \{l, h\}$ the realized state of nature. F observes it privately. However, it is commonly know that the "good" realization θ_l occurs with probability ν_1, if $a = 1$; with probability ν_0, if $a = 0$. As we refer to a PPP project, we can reasonably assume that exerting effort makes it more likely that the unit cost will be low: $0 < \nu_0 < \nu_1 < 1$. Exerting effort occasions to F a disutility of $\psi(0) = 0 < \psi(1) = \psi$.

Under complete information, in state i, the value of the future stream of profits at the beginning of period τ is given by $\Pi_{i,\tau} = \frac{\pi_i}{r} \left(1 - \frac{1}{(1+r)^{T_i - \tau}} \right)$, where r is the discount rate. The net present value of the project is:

$$\widetilde{\Pi}_i = \Pi_{i,0} - (I + \psi(a)).$$

Defining $w(q_i) \equiv S(q_i) - \theta q_i$, the discounted return of G from private management is given by $V_{i,\tau} = \frac{w(q_i)}{r} \left(1 - \frac{1}{(1+r)^{T_i - \tau}} \right) - \Pi_{i,\tau}$. Thus, the period$-0$ discounted return of G from private management amounts to:

$$V_{i,0} = \frac{w(q_i)}{r} \left(1 - \frac{1}{(1+r)^{T_i}} \right) - \Pi_{i,0}.$$

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[3]To be rigorous, spending one unit of public funds requires collecting more than one unit of money from taxpayers. To capture this circumstance formally, we could introduce some parameter $\lambda > 0$, expressing the shadow cost of public funds. Then, a transfer of t would cost $(1 + \lambda) t$ to G. However, because this would have no qualitative impact on results, we neglect the shadow cost of public funds, for simplicity.

The payoff obtained from the entire life of the project in state i is expressed as follows:

$$W_i = \frac{w(q_i^*)}{r} + \left(\frac{w(q_i)}{r} - \frac{w(q_i^*)}{r} \right) \left(1 - \frac{1}{(1+r)^{T_i}} \right) - \Pi_{i,0},$$

where q_i^* is the output level that maximizes $w(\cdot)$, defined by the marginal-cost pricing rule:

$$p(q_i^*) = \theta_i. \tag{1}$$

From now on, stars are appended to denote efficient values.

Provided that F faces a zero outside opportunity and that effort is desirable to G $(a^* = 1)$, an efficient allocation is one that solves the following programme, where we refer to $\Pi_{i,0}$, rather than to t_i, with a standard change of variable:[4]

$$\underset{\{q_i, \Pi_{i,0}; T_i\}_{i=l,h}}{Max} \mathbb{E}[W_i]$$

subject to

$$\mathbb{E}[\Pi_{i,0}] \geq I + \psi. \tag{2}$$

At optimum, $q_i = q_i^* \ \forall i$, and no surplus is left to F, so that optimized payoffs are given by:

$$\begin{aligned} \mathbb{E}\left[\Pi_{i,0}^*\right] &= I + \psi \\ \mathbb{E}\left[W_i^*\right] &= \frac{w(q_i^*)}{r} - (I + \psi). \end{aligned}$$

The termination dates T_l and T_h are irrelevant.

3 Information problems

There are two subsequent information problems, namely, moral hazard followed by adverse selection. First, since effort is not actually observable to G and it is costly to F, incentives must be provided for F to select the desirable level $a^* = 1$. Second, F observes the realization of θ privately. Applying the Revelation Principle, G makes a take-it-or-leave-it offer to F, which consists in the menu of contractual allocations $(\{q_l, \Pi_{i,0}; T_l\}, \{q_h, \Pi_{i,0}; T_h\})$. The peculiarity of our approach, relative to the literature on incentive contracts, is that we let the contractual length be conditioned on the state of nature. This, of course, encompasses the standard case of a fixed-term contract $(T_l = T_h = T)$. F picks one of the allocations in the menu after exerting

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[4]Effort is desirable when $\mathbb{E}[w(q_i^*)] - \widetilde{\mathbb{E}}[w(q_i^*)] > r\psi$, where $\mathbb{E}$ and $\widetilde{\mathbb{E}}$ are the expectation operators over the two states l and h corresponding, respectively, to $a = 1$ and $a = 0$.

effort a to construct the infrastructure and then learning the realization of θ; accordingly, it produces and it is compensated thereafter.

As a first step of analysis, we show that, while the contractual length is irrelevant under complete information, the presence of information problems on the firm's side may impose restrictions to the choice of the duration of a contract that stipulates an efficient allocation. Denoting $\Delta\theta = \theta_h - \theta_l$, $\Delta\nu = \nu_1 - \nu_0$ and $\Delta\Pi = \Pi_{l,0}^* - \Pi_{h,0}^*$, this requires identifying the pairs $\left\{\Pi_{i,0}^*, T_i^*\right\}_{i=l,h}$ such that the following *information constraints* are satisfied:

$$\Delta\Pi \geq \frac{\psi}{\Delta\nu} \tag{3}$$

$$\Delta\Pi \leq \Delta\theta q_l^* \sum_{\tau=0}^{T_l^*-1} \frac{1}{(1+r)^{\tau+1}} \tag{4}$$

$$\Delta\Pi \geq \Delta\theta q_h^* \sum_{\tau=0}^{T_h^*-1} \frac{1}{(1+r)^{\tau+1}}. \tag{5}$$

The moral-hazard constraint (3) ensures that the amount of risk $\Delta\Pi$, to which F is exposed if it operates in state h, rather than in state l, is large enough to motivate F to make state l more likely by exerting costly effort at the construction stage. The adverse-selection constraints (4) and (5) prevent F to announce, respectively, l in state h and h in state l. As from (4), a lie is prevented in state h if the benefit of $\Delta\Pi$, induced by that lie, does not exceed the penalty that F would incur by understating the cost, which is given by the difference between the true high cost and the announced low cost, in each production period through date T_l. As from (5), information is released in state l if the benefit $\Delta\Pi$, which F appropriates by reporting l rather than h, is at least as large as the gain that F would obtain by exaggerating the cost, which is given by the difference between the fake high cost and the true low cost, in each production period through date T_h.[5]

Provided that the information constraints are formulated in terms of $\Delta\Pi$, T_l^* and T_h^*, it is convenient to refer to the triplet $\left\{\Delta\Pi, T_l^*, T_h^*\right\}$ rather than to the pairs $\left\{\Pi_{i,0}^*, T_i^*\right\}_{i=l,h}$. This is possible because, when (2) holds as an equality, the profits of F can be expressed as a function

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[5]The formulation of the adverse-selection constraints in (4) and (5) is reminiscent of that which is found in repeated adverse-selection problems *à la* Baron and Besanko [1]. In those problems, private information is not persistent and the agent makes a new report to the principal in each subsequent period. In our model, the unit cost of operation is drawn once for all and the firm reports to the government only at date 0. However, a lie at that date can be assimilated to a repetition of the same lie, yielding the same output obligation and the same compensation right, in each subsequent period through the termination date. Essentially, what differs, in our setting, is that the number of periods, during which the firm could benefit from that lie, is endogenous to the contract.

of $\Delta\Pi$:

$$\Pi_{l,0}^* = I + \psi + (1 - \nu_1)\Delta\Pi \tag{6}$$

$$\Pi_{h,0}^* = I + \psi - \nu_1\Delta\Pi. \tag{7}$$

These expressions show that, apart from recovering the initial (monetary and non-monetary) investment $I + \psi$, F receives a "reward" of $(1 - \nu_1)\Delta\Pi$ in state l, and faces a "punishment" of $\nu_1\Delta\Pi$ in state h. Once $\Delta\Pi$ is chosen in compliance with (3), also (4) and (5) are met, provided that the contractual terms are set according to the following lemma.

Lemma 1 *The contract which stipulates an efficient allocation satisfies* $(3) - (5)$ *if and only if the triplet* $\{\Delta\Pi, T_l^*, T_h^*\}$ *is such that:*

$$\Delta\Pi \in \left[\frac{\psi}{\Delta\nu}, \frac{\Delta\theta q_l^*}{r}\right] \tag{8}$$

$$T_l^* \geq \underline{T}(\Delta\Pi) \equiv \frac{\ln \frac{\Delta\theta q_l^*}{\Delta\theta q_l^* - r\Delta\Pi}}{\ln(1+r)} \tag{9}$$

$$T_h^* \leq \overline{T}(\Delta\Pi) \equiv \frac{\ln \frac{\Delta\theta q_h^*}{\Delta\theta q_h^* - r\Delta\Pi}}{\ln(1+r)} \quad if \ \Delta\Pi < \frac{\Delta\theta q_h^*}{r}. \tag{10}$$

The condition $\Delta\Pi < \Delta\theta q_l^*/r$ is necessary for (4) to hold. If $\Delta\Pi \geq \Delta\theta q_h^*/r$, then (5) is satisfied, regardless of how T_h is set. Moreover, for (4) to hold together with (3), exerting effort must be not too costly to F, *i.e.*, we must have $\psi \leq \Delta\nu\Delta\theta q_l^*/r$. The main implication of the lemma is viewed by contrasting its content with the case of a fixed-term contract (Proposition 1 in Danau and Vinella [3]). In the presence of information problems, the range of feasible termination dates is narrower, if the contract is bound to have a fixed term of T, than is if the contract has a state-dependent duration. In particular, a fixed-term contract cannot have a duration shorter than $\underline{T}(\Delta\Pi)$. This is possible, instead, in the bad state when the contract has a state-dependent term. Imposing a time cut of $T - T_h$ does not make F more eager to understate the cost, because the penalty associated with that lie depends on T_l, rather than on T_h.

4 Limited commitment

Under limited commitment, the contract is exposed to the partners' opportunism during operation. This imposes further restrictions on contract design and, in particular, on the choice of the contractual length. After identifying these new restrictions, we will provide conditions under which the contract that stipulates an efficient allocation is robust to the partners' opportunism.

Either partner may renege on the contract in any state $i \in \{l, h\}$ and in any period $\tau \in \{0, ..., T_i - 1\}$. For the firm this incentive arises at the beginning of the period, before producing and incurring the associated cost. For the government it arises at the end of the period, when it is supposed to compensate the firm. Following renege, the two partners engage in a renegotiation game and may reach a new agreement, thus preventing the termination of the partnership. To identify the equilibrium of the game, we need to determine the payoff that each partner potentially obtains, if it reneges on the contract. This will depend, on the one hand, on the sanctions that a court of justice can impose to the partners; on the other, on the costs that break-up occasions.

Let us first consider the court of justice. When at least one of the parties is unwilling to execute the contract, the court cannot oblige them to do so. However, in the same vein as in the recent literature on non-enforceable contracts, we allow for the court to impose sanctions following a breach. Specifically, when G breaches the contract, the court can impose a penalty of $P \geq 0$ in favour of F, provided G appropriates (a part of) the private investment in that case. The penalty is limited because so is the enforcement ability of the court, due to an innate weakness of the judicial system or to legislative restrictions and voids. By contrast, there is no sanction for F if it terminates the partnership. This is because, in that case, break-up is assimilated to private default, involving that G relieves the SPV and the private partner cannot be called upon to contribute further resources, in addition to those already destined to construct the infrastructure. Because F renounces to recoup a part of its initial investment as it stops abiding by the contract, this loss will act naturally as a penalty, for the firm, in the event of a breach. Taking this into account, the PPP contract is complemented with a termination clause, according to which G will pay to F the penalty P if the PPP is terminated on its own initiative.[6] In addition to the sanction, the break-up occasions a cost of $R > 0$ to G. While this cost will represent an additional deterrent to renege for G, it will motivate F to renege, expecting G to be willing to renegotiate the contract. Therefore, existence of a break-up cost on the government's side is the root of possible renegotiation between G and F.

We hereafter present the outcome of the renegotiation game between G and F. This is derived assuming that renegotiation occurs under complete information; we show in Appendix B that this is actually the case in equilibrium.

Lemma 2 $\forall \tau, \tau' \in \{0, ..., T_i - 1\}$ *such that $\tau < \tau'$, if F reneges at τ and G believes that F will not renege at τ', then F does renege at τ'.*

The lemma suggests that, once F reneges at some date τ, G reasonably expects F to renege again at any $\tau' > \tau$. This result is essential to understand how much surplus will be shared

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[6]We introduce the breach penalty in the model for the sake of completeness. However, as will become apparent, this is not essential for results.

in the renegotiation game. Given that, following a renege at τ, renegotiation will occur in all periods through date $T_i - 1$, the available surplus is such that:

$$X_\tau + \frac{X_{\tau+1}}{1+r} + ... + \frac{X_{T_i-1}}{(1+r)^{T_i-\tau}} = R, \ \forall \tau \in \{0, ..., T_i - 1\}, \tag{11}$$

where $X_\tau, X_{\tau+1}, ..., X_{T_i-1}$ is, respectively, the surplus to be shared in period $\tau + 1, ..., T_i - 1$. Basically, (11) means that the maximum amount that F can extract from G, if they renegotiate repeatedly from date τ through date $T_i - 1$, sums up, in discounted terms, to the saving R that the continuation of the partnership grants to G. This is explained as follows. In each period τ, the firm knows that, if it abjures the contract, then the government will accept to renegotiate in order to avoid the break-up, which is costly. If there were no possibility of reneging again beyond date τ, then F could afford to force G to share its entire continuation benefit R. However, G is aware that, in the subsequent period, F will be motivated to breach also the new contract in order to take again advantage of the cost of break-up. In light of this, rather than sharing R with F repeatedly, G would prefer to terminate the partnership and incur that cost once for all. Being unable to convince G that it will not renege again beyond date τ, F accepts to renegotiate on a lower amount X_τ to avoid that outcome. Accordingly, a new agreement can be reached only if the parties negotiate on a surplus of $X_\tau < R$ at the end of period τ, $X_{\tau+1} < R$ at the end of $\tau + 1$, and so on, such that, overall, they share no more than the cost of break-up, in discounted terms.

Lemma 3 *Suppose that F reneges at τ. Under the reasonable belief that F will renege again in each subsequent period, the surplus to be shared in renegotiation amounts to $\frac{r}{1+r}R$ if $\tau < T_i - 1$, and to R if $\tau = T_i - 1$.*

Essentially, when F reneges before the last period of operation, the partners negotiate on a part $R - \frac{R}{1+r} = \frac{r}{1+r}R$ of the cost of break-up. The larger is the discount rate r, the less the firm can profit from future renegotiations, in discounted terms, hence the more surplus will be shared in period τ. There is one interesting implication to this result. Because F cannot take advantage of repeated renegotiation, the exact duration of the contract is irrelevant in this perspective. This explains why the expected profit of F from current and future renegotiations, in discounted terms, is independent of T_i. Indeed, it amounts to:

$$\Pi_{i,\tau}^{rn} = (1 - \alpha) R, \ \forall \tau,$$

where $1 - \alpha$ is the probability of F making a take-it or leave-it offer to G in the renegotiation game.

Let us now look at the behaviour of the government. If G reneges on the contract and the partnership is terminated, then G incurs not only the break-up cost but also the breach

10

penalty. Thus, if the partners renegotiate after G reneges, then F extracts a quota $(1 - \alpha)$ of the break-up cost R, as in the case where F itself reneges, together with the entire penalty P, which is the reservation utility of F when G reneges. This makes it onerous for G to renege repeatedly, which explains the following result.

Lemma 4 *Suppose that G reneges at τ. Then, it does not renege at any $\tau' > \tau$.*

Resting on this result, the earning of G, if it reneges on the contract in period τ, is determined as follows:
$$V_{i,\tau}^{rn} = \frac{w_i(q_i^*)}{r} \left(1 - \frac{1}{(1+r)^{T_i-\tau}} \right) - \frac{P + (1 - \alpha) R}{1 + r}, \ \forall \tau.$$

When renegotiating, no partner has an interest to propose a production level other than q_i^* (this result holds regardless of whether F or G reneges - see Appendix A). This is because a different production level would be inefficient for G and it would bring no benefit to F, hence it would only decrease the surplus to be shared in the renegotiation process. Thus, as in the contract, G appropriates the entire net surplus of the activity $\frac{w_i(q_i^*)}{r} \left(1 - \frac{1}{(1+r)^{T_i-\tau}} \right)$. This is diminished by the expected compensation that G owes to F, which is now given by $[P + (1 - \alpha) R] / (1 + r)$. This expression is understood if it considered that, should the partnership be terminated at the end of period τ, G would pay P to F and incur the cost R. In $V_{i,\tau}^{rn}$ these values are discounted at the beginning of the period.

To complete the picture, we point out that, once some partner reneges on the contract, the other partner has no interest to abjure the new deal, in turn (see Appendix B for the proof). This is intuitive in that no partner can touch a higher payoff than already obtained through the previous renegotiation. Moreover, assuming that each partner has zero outside opportunity, none of them prefers to terminate the contract in favour of that opportunity as long as:

$$\frac{w_h(q_h^*)}{r} \geq I + \psi \geq \frac{\psi}{\Delta \nu} \quad \text{and} \quad \frac{w_l(q_l^*)}{r} \geq \frac{P + (1 - \alpha) R}{1 + r} \tag{12}$$

First, under these conditions, the benefit G obtains from the project is sufficiently big relative to the cost it incurs for its realization as well as to the cost that it expects to bear if it reneges on the contract and the relationship is terminated. Second, the cost of investment is, in turn, sufficiently high to warrant that the profit $\Pi_{h,0}^*$ (as defined in (7)) is not too low to motivate F to terminate the relationship during operation. Henceforth, we take (12) to be satisfied so that the contract is not reneged upon unless some partner expects to reach a better deal within the relationship. Everything considered, for the contract to be renegotiation-proof, the constraints

$\Pi_{i,\tau}^* \geq \Pi_{i,\tau}^{rn}$ and $V_{i,\tau}^* \geq V_{i,\tau}^{rn}$ must be satisfied $\forall i, \tau$, where:

$$\Pi_{i,\tau}^* = \Pi_{i,0}^* \frac{(1+r)^{T_i} - (1+r)^{\tau}}{(1+r)^{T_i} - 1}$$

$$V_{i,\tau}^* = \frac{w_i(q_i^*)}{r}\left(1 - \frac{1}{(1+r)^{T_i-\tau}}\right) - \Pi_{i,\tau}^*.$$

Using these expressions, the constraints are respectively rewritten as:

$$T_i \leq \frac{\ln \frac{(1-\alpha)\frac{1+r}{r}R}{(1-\alpha)\frac{1+r}{r}R - \Pi_{i,0}^*}}{\ln(1+r)}, \; \forall i, \text{ if } (1-\alpha)\frac{1+r}{r}R \geq \Pi_{i,0}^* \tag{13}$$

$$\Pi_{l,0}^* \leq \frac{P + (1-\alpha)R}{1+r}. \tag{14}$$

First, (13) shows that the temptation of F to renege on the contract is related to its duration. As time goes by, the residual contractual profit $\Pi_{i,\tau}^*$ becomes smaller relative to the expected renegotiation profit $(1-\alpha)\frac{r}{1+r}R$. This makes it more difficult to motivate F to honour the contract through the termination date. The difficulty is exacerbated the longer is the contractual length because, in that case, the contractual profit is spread over a big number of periods. In addition, it is exacerbated in the bad state, in which the contractual profit of the firm is lower. Second, (14) shows that, on the opposite, the temptation of G to renege on the contract is unrelated to its duration. The reason is that the temptation of G arises as soon as the infrastructure is built and the state of nature becomes known, depending on the magnitude of the entire profit $\Pi_{i,0}^*$ G owes to F in the realized state i rather than on the residual profit $\Pi_{i,\tau}^*$. As $\Pi_{l,0}^* > \Pi_{h,0}^*$, G is more tempted to renege in the good state of nature, which explains why (14) is stated for $i = l$. Not surprisingly, P and R act as substitutes in the trade-off that G faces between complying with the initial contract and renegotiating. For G to be motivated to honour the contract, at least one between P and R must be sufficiently large. It involves that the contract that stipulates an efficient allocation might be renegotiation-proof, even if the judicial system is so weak that no penalty can be enforced ($P = 0$).

Coming back to the choice of T_l and T_h, the requirements dictated by the need to keep the opportunism of F under control during the execution of the contract (according to (13)) must be combined with those associated with the need to make the informational advantage of F innocuous early on in the relationship (as summarized in Lemma 1). To formalize this result, it is useful to define:

$$T_h^{lc}(\Delta\Pi) \equiv \frac{\ln \frac{(1-\alpha)\frac{1+r}{r}R}{(1-\alpha)\frac{1+r}{r}R - (I+\psi-\nu_1\Delta\Pi)}}{\ln(1+r)}. \tag{15}$$

This is the contractual length such that (13) holds as an equality in state h, provided that

$(1 - \alpha) \frac{1+r}{r} R > I + \psi - \nu_1 \Delta\Pi.$

Proposition 1 *Assume that:*

$$P \geq (1 + r) \left[I + \psi + (1 - \nu_1) \frac{\psi}{\Delta\nu} \right] - (1 - \alpha) R. \tag{16}$$

There exists a triplet $\{\Delta\Pi, T_l^*, T_h^*\}$ such that the contract stipulating an efficient allocation is renegotiation-proof if and only if:

$$(1 - \alpha) \frac{1 + r}{r} R \leq \frac{I + \psi + (1 - \nu_1) \frac{\psi}{\Delta\nu}}{\frac{\psi}{\Delta\nu}} \frac{\Delta\theta q_l^*}{r}. \tag{17}$$

When

$$(1 - \alpha) \frac{1 + r}{r} R \leq \frac{I + \psi - \nu_1 \frac{\psi}{\Delta\nu}}{\frac{\psi}{\Delta\nu}} \frac{\Delta\theta q_l^*}{r}, \tag{18}$$

$\exists \Delta\Pi \in \left[\frac{\psi}{\Delta\nu}, \frac{\Delta\theta q_l^*}{r} \right]$ *such that the contract is renegotiation-proof if $T_l^* = T_h^* \in \left[\underline{T}(\Delta\Pi), T_h^{lc}(\Delta\Pi) \right]$;*
when

$$\frac{I + \psi - \nu_1 \frac{\psi}{\Delta\nu}}{\frac{\psi}{\Delta\nu}} \frac{\Delta\theta q_l^*}{r} < (1 - \alpha) \frac{1 + r}{r} R \leq \frac{I + \psi + (1 - \nu_1) \frac{\psi}{\Delta\nu}}{\frac{\psi}{\Delta\nu}} \frac{\Delta\theta q_l^*}{r}, \tag{19}$$

the contract is renegotiation-proof only if $T_h^* \leq T_h^{lc}(\Delta\Pi) < \underline{T}(\Delta\Pi) \leq T_l^*$, $\forall \Delta\Pi \in \left[\frac{\psi}{\Delta\nu}, \frac{\Delta\theta q_l^*}{r} \right]$.

The core lesson conveyed by the proposition is that, for an efficient allocation to be decentralized by means of a renegotiation-proof contract, it might be necessary to differentiate the length of the contract between states of nature. This is actually the case when the expected renegotiation profit of F, namely $(1 - \alpha) \frac{1+r}{r} R$, is large enough to meet (19), involving that F is strongly tempted to breach the contract during its execution. Essentially, the contract must have a longer duration in the good state than in the bad state: $T_l^* > T_h^*$. The need to shorten the relationship in state h, relative to state l, reflects the difficulty, in state h, of inducing the firm first to release information and then to abide by the contractual obligations, which arise accordingly, through the termination date. While T_l must be set high enough to make the fake report θ_l unattractive, T_h must be set low enough to eliminate the temptation to renege during the operation phase. In addition, this must all be reconciled with the need to transfer risk to F so as to prevent shirking in construction, which requires creating a sufficiently large wedge between profits in the two states of nature.

To enucleate the specific impact of moral hazard and adverse selection on the possibility of making the contract renegotiation-proof, it is useful to restate (18) as follows:

$$(1 - \alpha) \frac{1 + r}{r} R \leq \frac{1}{\frac{\Pi_{l,0}^* - \Pi_{h,0}^*}{\Pi_{h,0}^*}} \frac{\Delta\theta q_l^*}{r}. \tag{20}$$

13

First, this condition is tighter the higher is the ratio $\left(\Pi_{l,0}^* - \Pi_{h,0}^*\right)/\Pi_{h,0}^*$ at which the profit grows from the bad state to the good state. As this rate is lowest when (3) is binding $\left(\Pi_{l,0}^* - \Pi_{h,0}^* = \psi/\Delta\nu\right)$, it is clear that the extent to which (20) can be relaxed depends finely on the severity of the moral-hazard problem. If exerting effort were costless, then profits could be set equal between states and (20) would hold trivially. That is, it would not be necessary to condition the contractual duration on the cost of production to make the contract renegotiation-proof. The more costly is effort, the more likely it is that renegotiation-proofness calls for $T_l^* > T_h^*$. Second, (20) is tighter the smaller is the ratio $\Delta\theta q_l^*/r$. This is the value at date 0 of the penalty that F faces if it claims l in state h at the outset of the operation phase in a contract with infinite duration, hence the largest possible penalty that F can incur following a lie in the bad state. As this penalty becomes smaller, it is more difficult to induce truthtelling in the bad state. A state-dependent contractual policy is then necessary.

5 From fixed to state-dependent duration: a flexibility gain

From Lemma 1 we learnt that, given a certain amount of risk $\Delta\Pi$ to be transferred to F, G enjoys more flexibility at adjusting the two termination dates T_l and T_h than a fixed term T. This is because the possibility of compensating adjustments in the contractual terms with adjustments in the per-period profits of F is enhanced. We shall now formalize the "substitutability" between contractual terms and per-period profits for a rigorous determination of the flexibility gain.

For the profit wedge to be such that (4) and (5) are satisfied, it must take the following expression:

$$\Delta\Pi = \frac{\Delta\theta z_j}{r}\left(1 - \frac{1}{(1+r)^{T_j}}\right), \tag{21}$$

where $j \in \{l, h\}$, not necessarily coinciding with the true state i, and with the additional requirement that $\sum_{\tau=0}^{T_l-1}\frac{z_l}{(1+r)^{\tau+1}} = \sum_{\tau=0}^{T_h-1}\frac{z_h}{(1+r)^{\tau+1}}$. Expressed in this way, $\Delta\Pi$ denotes the cumulated discounted wedge between the per-period rewards and punishments to be faced by F through the termination date. This expression of the wedge is "normalized" as if that date were either T_l in *both* states or T_h in *both* states. Specifically, for $j = l$, (21) means that a per-period reward of $(1 - \nu_1)\Delta\theta z_l$ is granted to F for T_l periods, where $z_l \in \left(\frac{r\psi}{\Delta\nu\Delta\theta}, q_l^*\right]$; for $j = h$, it means that a per-period punishment of $\nu_1\Delta\theta z_h$ is inflicted to F for T_h periods, where $z_h \in [q_h^*, \infty)$. When deciding about $\Delta\Pi$ and T_l, G is basically choosing z_l, hence the per-period reward in a contract with duration T_l. Symmetrically, when deciding about $\Delta\Pi$ and T_h, G is choosing z_h, hence the per-period punishment in a contract with duration T_h.

Corollary 1 *Assume that* (16) *and* (17) *are satisfied. If* $T_h^{lc}\left(\frac{\psi}{\Delta\nu}\right) < \underline{T}\left(\frac{\psi}{\Delta\nu}\right),$ *then the contract which stipulates an efficient allocation is made renegotiation-proof by setting* z_h *above* q_l^* :

$$z_h \geq \frac{(1-\alpha)\frac{1+r}{r}R\frac{\psi}{\Delta\nu}}{(I+\psi-\nu_1\Delta\Pi)\frac{\Delta\theta}{r}} > q_l^*.$$

By shortening the duration T_h below $\underline{T}\left(\frac{\psi}{\Delta\nu}\right)$, G can raise the per-period punishment above the maximum value of $\nu_1\Delta\theta q_l^*$, which is feasible with a fixed term in compliance with the adverse-selection constraints, without exposing the firm to more risk than is strictly necessary to solve the moral-hazard problem.

6   Concluding remarks

There are essentially two lessons on PPP contracts to be drawn from our analysis. First, in line with the findings on flexible-term contracts, what causes a contract with a state-dependent duration to perform better than a fixed-term contract is the possibility of the firm behaving opportunistically *vis-à-vis* the government during its execution. This result holds true despite that not only the firm but also the government lacks the ability to commit. Second, the reason why the partnership should have a shorter duration when the operating conditions are unfavourable, rather than favourable, is that, when the former conditions are realized, the firm can be punished more in operation than can be under a fixed-term contract. This is due to the enhanced substitutability between per-period punishment and contractual length, which enables the government to raise the per-period punishment beyond the limits of the fixed-term contract. Therefore, for incentive purposes, it is optimal to set a low profit and a short duration in bad states of nature. The discrepancy between this contractual approach and the flexible-term proposal evidences the need for a better understanding of the interaction between contractual non-enforceability and contractual incompleteness as distinct sources of limited commitment. For an optimal design of PPP contracts, it would thus be useful to investigate further in that direction.

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A Limited commitment

Suppose that some party reneges at date τ in state i. The partners return to the contracting table. Renegotiation may either fail or succeed.

Break-up payoffs   Suppose that renegotiation fails and break-up follows. F obtains $\Pi_{i,\tau}^{b,G} = P$, if G reneges; it obtains $\Pi_{i,\tau}^{b,F} = 0$, if F itself reneges. G obtains:

$$V_{i,\tau}^{b,F} = \frac{w_i\left(q_i^*\right)}{r}\left(1 - \frac{1}{\left(1+r\right)^{T_i-\tau}}\right) - R,$$

if F reneges at the beginning of period τ; it obtains:

$$V_{i,\tau}^{b,G} = \frac{w_i\left(q_i^*\right)}{r}\left(1 - \frac{1}{\left(1+r\right)^{T_i-\tau}}\right) - \frac{R+P}{1+r},$$

if G itself reneges at the end of τ.

Renegotiation payoffs   Suppose that renegotiation succeeds. The pair of variables $\left\{q_i^{ar}, \Pi_{i,\tau}^{ar}\right\}$, on which the partners renegotiate, replaces the pair $\left\{q_i^*, \Pi_{i,\tau}^*\right\}$ stipulated in the contract. F obtains $\Pi_{i,\tau}^{ar}$; G obtains $V_{i,\tau}^{ar} = \frac{w_i\left(q_i^{ar}\right)}{r}\left(1 - \frac{1}{\left(1+r\right)^{T_i-\tau}}\right) - \Pi_{i,\tau}^{ar}$.

A.1   Proof of Lemma 2

Suppose that F reneges at τ and that G believes that F will not renege again in the future. The surplus to be shared in renegotiation is R. With probability $\left(1 - \alpha\right)$, F makes a take-it-or leave-it offer to G, according to which G is left with $V_{i,\tau}^{b,F}$. The payoff of F from renegotiation is $\Pi_{i,\tau}^{ar} = \frac{w_i\left(q_i^{ar}\right)}{r}\left(1 - \frac{1}{\left(1+r\right)^{T_i-\tau}}\right) - V_{i,\tau}^{b,F}$. This is highest for $q_i^{ar} = q_i^*$. With probability α, G makes a take-it or leave-it offer to F, according to which F is left with $\Pi_{i,\tau}^{b,F} = 0$. Therefore, the expected payoff of F from renegotiation is $\Pi_{i,\tau}^{rn} = \left(1 - \alpha\right)R$. The per-period profit π_τ^{rn} that F would obtain under the new agreement is such that:

$$\Pi_{i,\tau}^{rn} = \frac{\pi_\tau^{rn}}{r}\left(1 - \frac{1}{\left(1+r\right)^{T-\tau}}\right). \tag{22}$$

Using $\Pi_{i,\tau}^{rn} = \left(1 - \alpha\right)R$ in (22), the residual profit of F at any $\tau' > \tau$ is:

$$\widetilde{\Pi}_{i,\tau',\tau} = \frac{\pi_\tau^{rn}}{r}\left(1 - \frac{1}{\left(1+r\right)^{T-\tau'}}\right) = \left(1 - \alpha\right)R\left(1+r\right)^{\tau'-\tau}\frac{\left(1+r\right)^{T-\tau'} - 1}{\left(1+r\right)^{T-\tau} - 1}.$$

If F reneges again at τ', it gets:

$$\Pi_{i,\tau'}^{rn} = \Pi_{i,\tau}^{rn} = \left(1 - \alpha\right)R > \left(1 - \alpha\right)R\left(1+r\right)^{\tau'-\tau}\frac{\left(1+r\right)^{T-\tau'} - 1}{\left(1+r\right)^{T-\tau} - 1} = \widetilde{\Pi}_{i,\tau',\tau}.$$

Therefore, the belief that F will not renege at τ' does lead to F reneging again at τ'.

A.2   Proof of Lemma 3

Using (11), the following results are obtained by backward induction:

$$X_{i,T_i-1} = R$$

$$X_{i,T_i-2} = R - \frac{X_{i,T_i-1}}{1+r} = R - \frac{R}{1+r}$$

$$X_{i,T_i-3} = R - \frac{X_{i,T_i-2}}{1+r} - \frac{X_{i,T_i-1}}{(1+r)^2} = R - \frac{R}{1+r}$$

$$...$$

$$X_{i,T_i-\tau} = R - \frac{X_{i,T_i-\tau+1}}{1+r} - \frac{X_{i,T_i-\tau+2}}{(1+r)^2} - ... - \frac{X_{i,T_i-1}}{(1+r)^{\tau+1}} = R - \frac{R}{1+r}.$$

A.3   Proof of Lemma 4

Suppose that G reneges at the end of period τ. With probability α, it makes a take-it or leave-if offer to F, according to which F is left with $\Pi_{i,\tau}^{b,G} = P$. G earns $V_{i,\tau}^{ar} = \frac{w_i(q_i^{ar})}{r}\left(1 - \frac{1}{(1+r)^{T_i-\tau}}\right) - P$. This is highest for $q_i^{ar} = q_i^*$. With probability $(1-\alpha)$, F makes a take-it-or-leave-it offer to G, according to which G is left with $V_{i,\tau}^{b,G}$. F obtains $\Pi_{i,\tau}^{ar} = \frac{w_i(q_i^{ar})}{r}\left(1 - \frac{1}{(1+r)^{T_i-\tau}}\right) - V_{i,\tau}^{b,F}$, which is highest for $q_i^{ar} = q_i^*$. Then, the expected payoff of G if it reneges at τ and never beyond that date is given by:

$$V_\tau^{rn} = \frac{w_i(q_i^*)}{r}\left(1 - \frac{1}{(1+r)^{T-\tau}}\right) - \frac{P + (1-\alpha)R}{1+r}.$$

The per-period benefit of G is the value of $\upsilon_{i,\tau}$ satisfying:

$$V_\tau^{rn} = \frac{\upsilon_{i,\tau}}{r}\left(1 - \frac{1}{(1+r)^{T-\tau}}\right).$$

The residual return of G at $\tau' > \tau$ is given by:

$$\begin{aligned}
\widetilde{V}_{\tau',\tau}^{rn} &= \frac{\upsilon_{i,\tau}}{r}\left(1 - \frac{1}{(1+r)^{T-\tau'}}\right) \\
&= V_\tau^{rn}\frac{(1+r)^{T-\tau'} - 1}{(1+r)^{T-\tau'}}\frac{(1+r)^{T-\tau}}{(1+r)^{T-\tau} - 1} \\
&= \frac{w_i(q_i^*)}{r}\frac{(1+r)^{T-\tau'} - 1}{(1+r)^{T-\tau'}} - \frac{P + (1-\alpha)R}{1+r}\frac{(1+r)^{T-\tau'} - 1}{(1+r)^{T-\tau'}}\frac{(1+r)^{T-\tau}}{(1+r)^{T-\tau} - 1}.
\end{aligned}$$

If G reneges at τ', then it will have to give up again an expected profit of $[P + (1 - \alpha) R] / (1 + r)$, thus obtaining $V_{\tau'}^{rn}$ instead of $\widetilde{V}_{\tau',\tau}^{rn}$. Because:

$$\widetilde{V}_{\tau',\tau}^{rn} > V_{\tau'}^{rn} \Leftrightarrow \frac{P + (1 - \alpha) R}{1 + r} \left(1 - \frac{(1 + r)^{T - \tau'} - 1}{(1 + r)^{T - \tau'}} \frac{(1 + r)^{T - \tau}}{(1 + r)^{T - \tau} - 1} \right) > 0,$$

G prefers $V_{\tau'}^{rn}$ to $\widetilde{V}_{\tau',\tau}^{rn}$.

B   Proof of Proposition 1

We first prove conditions (16) - (19). We then show that no partner reneges following a renege by the other partner. As a final step, we prove that cheating off the equilibrium path (*i.e.*, misrepresentation of θ_i anticipating renege at τ) is not an issue.

B.1   Proof of (16) - (19)

Take $T_l = T_h = T$. From Lemma 1, $\Pi_{l,0}^* > \Pi_{h,0}^*$. Using (7), (13) specifies as $T \leq T_h^{lc}(\Delta\Pi)$. Using the definitions of $T_h^{lc}(\Delta\Pi)$ and $\underline{T}(\Delta\Pi)$, we check that:

$$T_h^{lc}(\Delta\Pi) \geq \underline{T}(\Delta\Pi) \Leftrightarrow (1 - \alpha) \frac{1 + r}{r} R \leq \frac{I + \psi - \nu_1 \Delta\Pi}{\Delta\Pi} \frac{\Delta\theta q_l^*}{r}. \tag{23}$$

This condition is weakest when $\Delta\Pi$ takes the lowest feasible value in (8): $\Delta\Pi = \psi/\Delta\nu$. Accordingly, (23) is rewritten as (18). Suppose now that (18) is violated. From (9) and $T \leq T_h^{lc}(\Delta\Pi)$, it follows: $T_h \leq T_h^{lc}(\Delta\Pi) < \underline{T}(\Delta\Pi) \leq T_l$.

We are left with checking (13) in state l :

$$T_l \leq T_l^{lc}(\Delta\Pi) \equiv \frac{\ln \frac{(1 - \alpha)\frac{1 + r}{r} R}{(1 - \alpha)\frac{1 + r}{r} R - [I + \psi + (1 - \nu_1)\Delta\Pi]}}{\ln(1 + r)}.$$

$\exists T_l$ satisfying $\underline{T}(\Delta\Pi) \leq T_l$ and $T_l \leq T_l^{lc}(\Delta\Pi)$ if and only if:

$$\underline{T}(\Delta\Pi) \leq T_l^{lc}(\Delta\Pi) \Leftrightarrow (1 - \alpha) \frac{1 + r}{r} R \leq \frac{I + \psi + (1 - \nu_1)\Delta\Pi}{\Delta\Pi} \frac{\Delta\theta q_l^*}{r}.$$

This condition is weakest when $\Delta\Pi = \psi/\Delta\nu$. Accordingly, it is rewritten as (17).

Using (6) in (14), we see that the latter is weakest when $\Delta\Pi = \psi/\Delta\nu$. Accordingly, it is rewritten as (16).

B.2   No partner reneges following a renege by the other partner

As $P + (1 - \alpha) R \geq (1 - \alpha) R$, F gains less if it reneges at the beginning of period $T_i - 1$ following a renege by G at the end of period $T_i - 2$.

As $[P + (1 - \alpha) R] (1 + r) > (1 - \alpha) [R - R/(1 + r)]$, F gains less if it reneges at the beginning of any $\tau \in \{1, ..., T_i - 2\}$ following a renege by G at the end of period $\tau - 1$.

The two above inequalities further imply that, if G reneges at the end of any period $\tau \in \{0, ..., T_i - 1\}$, then it must concede to F a higher profit than if F reneges at the beginning of period τ.

B.3 Information constraints anticipating a renege

Let $\Pi_{i,\tau}^{RN}$ denote the payoff that F would obtain in state i, discounted at time τ, if it were to cheat at the outset of the operation phase and renege were to occur at τ. We only consider the case where F might renege at τ, provided the case of G reneging is analogous. Also let $\pi_{i,x}^*$ be the instantaneous profit in state i in period $x \in \{0, ..., \tau\}$. F has no incentive to lie if and only if:

$$\Pi_{l,0}^* \geq \sum_{x=0}^{\tau} \frac{\left(\pi_{h,x}^* + \Delta\theta q_h^*\right)}{\left(1+r\right)^{x+1}} + \Pi_{l,\tau}^{RN} \tag{24a}$$

$$\Pi_{h,0}^* \geq \sum_{x=0}^{\tau} \frac{\left(\pi_{l,x}^* - \Delta\theta q_l^*\right)}{\left(1+r\right)^{x+1}} + \Pi_{h,\tau}^{RN}. \tag{24b}$$

We hereafter show that (24a) is satisfied. If F reports h at date 0, in state l, and the contract is renegotiated at some $\tau \in \{0, ..., T_h - 1\}$, the residual profit of F corresponding to the period $\{\tau, ..., T_h - 1\}$ is:

$$\Pi_{l,\tau}^{RN} = \Pi_{h,\tau}^{rn} + \sum_{x=\tau}^{T_h-1} \frac{\Delta\theta q_h^*}{\left(1+r\right)^{T_h-x}}$$

where $\Pi_{h,\tau}^{rn}$ is the profit that a type h can extract in case of renegotiation. Hence:

$$\Pi_{l,\tau}^{RN} = \sum_{x=\tau+1}^{T_h} \frac{\pi_{l,\tau}^{RN}}{\left(1+r\right)^{T_h-x}} = \Pi_{h,\tau}^{rn} + \sum_{x=\tau+1}^{T_h} \frac{\Delta\theta q_h^*}{\left(1+r\right)^{T_h-x}}.$$

(24a) becomes:

$$\begin{aligned}
\Pi_{l,0}^* \geq{} & \Pi_{h,0}^* + \sum_{x=1}^{T_h} \frac{\Delta\theta q_h^*}{\left(1+r\right)^x} \\
& + \frac{1}{\left(1+r\right)^\tau} \left[\Pi_{h,\tau}^{rn} + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q_h^*}{\left(1+r\right)^{T_h-x}} - \left(\Pi_{h,\tau}^* + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q_h^*}{\left(1+r\right)^{T_h-x}} \right) \right].
\end{aligned} \tag{25}$$

Recalling that (13) in state h is equivalent to $\Pi_{h,\tau}^* \geq \Pi_{h,\tau}^{rn}$, we see that:

$$\Pi_{h,\tau}^* + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q_h^*}{\left(1+r\right)^{T_h-x}} \geq \Pi_{h,\tau}^{rn} + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q_h^*}{\left(1+r\right)^{T_h-x}}.$$

Hence, (24a) is implied by (5) and (13).

Symmetrically, (24b) is implied by (4) and (13).

C   Proof of Corollary 1

For $j = h$ and $\Delta\Pi = \frac{\psi}{\Delta\nu}$ (21) is written as:

$$\frac{\psi}{\Delta\nu} = \frac{\Delta\theta z_h}{r}\left(1 - \frac{1}{(1+r)^{T_h}}\right) \Leftrightarrow T_h = \frac{\ln\frac{\Delta\theta z_h}{\Delta\theta z_h - r\frac{\psi}{\Delta\nu}}}{\ln(1+r)}$$

Hence:

$$T_h \leq T_h^{lc}\left(\frac{\psi}{\Delta\nu}\right) \Leftrightarrow z_h \geq \frac{(1-\alpha)\frac{1+r}{r}R\frac{\psi}{\Delta\nu}}{(I + \psi - \nu_1\Delta\Pi)\frac{\Delta\theta}{r}}$$

Moreover:

$$T_h^{lc}\left(\frac{\psi}{\Delta\nu}\right) < \underline{T}\left(\frac{\psi}{\Delta\nu}\right) \Leftrightarrow \frac{(1-\alpha)\frac{1+r}{r}R\frac{\psi}{\Delta\nu}}{(I + \psi - \frac{\nu_1}{\Delta\nu}\psi)\frac{\Delta\theta}{r}} > q_l^*.$$

The result follows.