Equal opportunities in school choice

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Motivation

- Equality of opportunity (EOp) - *(Roemer 1998, Fleurbaey, 2008)*
- Education key for EOp - *(Roemer and Unveren, 2017; Corak, 2013)*
- Typical policy recommendation: investment to improve quality.
- What about the students' allocation? (Complementary policy)
- Public school choice is a many-to-one matching market: no price

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Example: Student proposing Deferred Acceptance

New York City assigns students to Highs school using Student proposing Deferred Acceptance (Gale and Shapley, 1962)

- $\mathbf{i_1}, \mathbf{i_2}, \mathbf{i_3} : s_1 \; P \; s_2$
- $\mathbf{i_4} : s_2 \; P \; s_1$

- $\mathbf{s_1} : i_4 \succ i_3 \succ i_2 \succ i_1; \qquad q = 1$
- $\mathbf{s_2} : i_1 \succ i_2 \succ i_3 \succ i_4; \qquad q = 3$

- Round 1:
  - $s_1 \leftarrow (\not{i_1}, \not{i_2}, i_3)$
  - $s_2 \leftarrow (i_4)$
- Round 2:
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Affirmative Action and Match Quality

- Distributional concern in the matching theory literature: affirmative action.
- Affirmative action $\neq$ EOp
  - Who is loosing? (Students for Fair Admissions, Inc. v. President and Fellows of Harvard College.)
  - Weakening stability
- Preferences $\neq$ Quality (*Abdulkadiroglu et al. 2021*)

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Framework

The school choice setting

- $\mathcal{I} = \langle I, S, P, \succ, q, T, \lhd, U \rangle$
- $I = \{i_1, i_2, ..., i_{|I|}\}$students
- $S = \{s_1, s_2, ..., s_{|S|}\}$ schools
- $P= (P_{i_1}, ..., P_{i_{|I|}})$ students' preferences profile (strict and complete linear orders) over S
- $\succ= (\succ_{s_1}, ..., \succ_{s_{|S|}})$ schools' priority
- $q = (q_{s_1}, ..., q_{s_{|S|}})$ quota profile of schools
- $T = \{t_1, t_2, ..., t_{|T|}\}$ set of types (or type partition)
- $\lhd$ complete pre-order of types s.t. $t \lhd t'$ if students of type t are less advantaged (or have stronger needs) than those of t'.
- Match quality

$$\begin{bmatrix} U(i_1, s_1) & ... & U(i_1, s_{|S|}) \\ ... & ... & ... \\ U(i_{|I|}, s_1) & ... & U(i_{|I|}, s_{|S|}) \end{bmatrix}$$

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Matchings

- An assignment is a function $\mu : I \cup S \to 2^{I \cup S}$ such that:
  - for all $i \in I$, $\mu(i) \in S \setminus \emptyset$;
  - for all $s \in S$, $|\mu(s)| \leq q_s$ and $\mu(s) \subseteq I$;
  - $\mu(i) = s$ if and only if $i \in \mu(s)$
- **Stability:** there exist no student-school pair preferring each other to their current match.

No blocking pairs:

- $\mathcal{M}^\star \neq \emptyset$ (Gale and Shapley, 1962) and finite
- BUT enumerating stable assignments is NP-complete (Irving and Leather, 1986)

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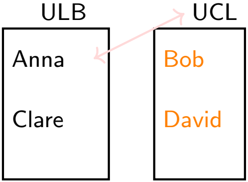

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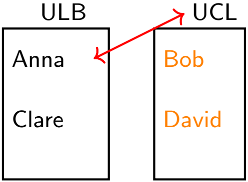

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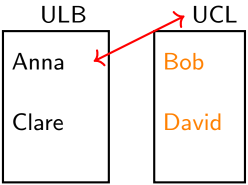

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Desirable requirements

- Efficiency: exhaust Paretian improvements of MQ

U(Anna, ULB) = U(Anna, UCL)
U(David, ULB) > U(David, UCL)

- Fairness: exhaust Pigou-Dalton transfers of MQ

U(Clare, UCL) = U(Clare, ULB) + **10**
U(Bob, ULB) = U(Bob, UCL) - **10**

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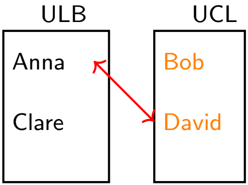

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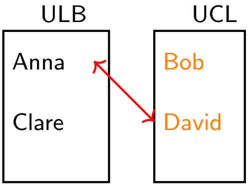

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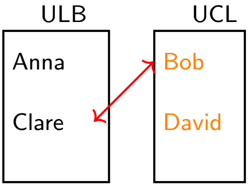

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Social welfare functions

- Call $W : \mathcal{M}^{\star} \to \mathbb{R}$ the social welfare associated to a stable school assignment.
- (Peragine, 2004)

$$W(\mu) = \sum_{i \in I} \omega_i U(i, \mu(i))$$

  such that
  - for all $i, i' \in t$, $\omega_i = \omega_{i'} > 0$; and
  - for all $i \in t$, $i' \in t'$ and $t, t' \in T$, if $t \lhd t'$, then $\omega_i > \omega_{i'}$.

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The SOE algorithm

- The Stable Opportunity Egalitarian (SOE) algorithm maximizes Social Welfare within the set of stable matchings
- Overcomes NP-completeness, efficient running time
- Realizes Affirmative Actions under standard stability notions
- Non technical intuition:
  - Start from Student optimal Deferred Acceptance
  - Move to another stable allocation (*reallocate students*) only if it improves Social Welfare

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Application

Empirical strategy

- A simulation based on INVALSI data (2021-2022)
- The smallest Italian province: Trieste (212.5 km^2).
- High school tracks: Scientific (549 students - 4 schools), Humanitarian (161 students - 2 schools), Technical (363 students - 4 schools) and Professional (81 students - 2 schools).
- 5 groups according to Economic, Social and Cultural Status (ESCS)

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Match quality and preferences

- Match quality: average performance of similar students
- $U(i, s) = \mathbb{E}\left[Score_j | \mu(j) = s, j \in t(i)\right]$
- Simulate (500 times) preferences of students and schools.
- School currently attended is always top ranked in student preference
- We compute

$$W(\mu) = \sum_{i \in I} \omega_i U(i, \mu(i))$$

  with $\omega_i \in \{2, 1.6, 1.4, 1.2, 1\}$
- We compare with W of current allocation

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Results - Social Welfare

Table: Social welfare difference with respect to Student optimal Deferred Acceptance

Assignment	Scientific	Humanitarian	Technical	Professional
Efficient and Fair	4.76 ***	2.16 ***	5.84 ***	2.31 ***
	(0 , 35)	(0 , 13)	(0 , 40.4)	(0 , 17.2)

Description: The table reports the average difference between social welfare in the considered matching and in Student optimal DA. Minimum and Maximum in parenthesis. P-values: * ($p < 0.10$), ** ($p < 0.05$), *** ($p < 0.01$).

Source: Own elaboration.

Results - Inequality of opportunity

Table: Inequality of opportunity difference with respect to Student optimal Deferred Acceptance

Assignment	Scientific	Humanitarian	Technical	Professional
Efficient and Fair	-0.02 ***	0 ***	-0.02 ***	0 **
	(-0.09 , 0.02)	(0 , 0.05)	(-0.11 , 0.1)	(-0.07 , 0.13)

Description: The table reports the average difference between relative IOp in the considered matching and in Student optimal DA. Minimum and Maximum in parenthesis. P-values: * ($p < 0.10$), ** ($p < 0.05$), *** ($p < 0.01$).

Source: Own elaboration.

Result - Frequency Student Optimal

Table: Probability of Student optimal Deferred Acceptance equal to Assignment.

Assignment	Scientific	Humanitarian	Technical	Professional
Efficient and Fair	0.492	0.422	0.392	0.436

Description: The table reports the share of times (out of 500) Student optimal DA has been found to solve the above allocative problems.

Source: Own elaboration.

Conclusion

- Merge Equality of Opportunity and Matching Theory.
- New concept of fairness in school choice settings.
- Define the problem as maximization of a Social Welfare function under stability
- The SOE algorithm solves it efficiently
- Simulation shows potentials for practical implementation
- Many more applications of SOE:
  - doctor-hospital;
  - refugees allocation;
  - equalizing hospital's opportunity for good doctors, to reduce regional disparities in the health care system;
  - any many-to-one matching problem which can be expressed as maximization of a linear social welfare function over the set of stable matchings

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