Comparative Skill Premia

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Abstract

This paper studies the effect of factor proportions and factor intensity on the skill premium. The model features a Heckscher-Ohlin structure augmented with non-neutral firm heterogeneity and matching and screening costs. The main result is that the skill premium is higher in the skill intensive industry (within a country) and maybe higher in the skill abundant country. This results contrast with the traditional Heckscher-Ohlin model (with trade cost) where the skill premium is the same in all industries and is lower in the skill abundant country. Interestingly, this result stems precisely from Heckscher-Ohlin mechanisms interacting with matching and screening costs and with non-neutral heterogeneity.

J.E.L. Classification. F1.

1 Notice

This is a preliminary version. The introduction and conclusion are to be completed and revised extensively. The review of the literature is to be updated and better organised.

2 Introduction

The objective of this paper is to study the relationship between factor price and firm-country-industry characteristics. To this purpose I build a simple model that features only two key elements of trade theory: countries differ by relative endowments, industries *and firms* differ by factor intensity.

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The model structure is constituted by two-country, two differentiated goods, two heterogeneous factors, a continuum of heterogeneous firms, search and screening costs, and wage determination at firm level. The variable of interest is the skill premium defined as the wage of skilled labor over the wage of unskilled labor at firm level. I compare the skill premium across firms, industries and countries. Since firms are heterogenous, when I compare the skill premium between firms I compare firms that would be identical if they did not belong to different industries or countries. I refer to any two such firms as twin firms.

The main result may be condensed as follows:

For any twin firms the skill premium is larger in the skill intensive industry and may be larger in the skill abundant country. The same applies to the industry-average skill premium.

This results contrast with the traditional Heckscher-Ohlin model (with trade cost) where the skill premium is the same in all industries and is lower in the skill abundant country. It contrast as well with the results of models that integrate Hicks-neutral firm heterogeneity in the Heckscher-Ohlin model; in such models factor prices behave exactly as in the traditional Heckscher-Ohlin model. The result above does not come out of magic, however. Interestingly, it stems precisely from the HO mechanisms: factor proportions and factor intensity, interacting with non-neutral firm heterogeneity, give rise to the result above. Furthermore, reservation wages do respond to factor proportions in the usual way (lower relative reservation wage for skilled workers in the skill abundant country); but the effect of reservation wages on matching and screening costs and the interaction with non neutral heterogeneity makes that the skill premium is larger in the skill intensive industry and may be larger in the skill abundant country.

In terms of the model structure the present paper relates to the literature on trade integration and the skill premium.[1] I use many of the elements present in that literature: firm and factor heterogeneity, search and screening costs, monopolistic competition, to mention only a few. But there is an important difference with that literature in terms of model structure. In the model I present country-industry characteristics interact with firms characteristics to give rise to the results. There are other differences in the model structure. Most of the literature on trade integration and the skill prem<ium

[1] See, e.g., Yeaple (2005), Chaney (2008), Melitz and Ottaviano (2008), Arkolakis, Costinot and Andrés Rodríguez (2012), Bustos (2011), Eaton, Kortum and Kramarz (2011), Crozet, Head and Mayer (2012). See also Manasse and Turrini (2001), Egger and Kreickemeier (2009), Helpman, Itskhoki and Redding (2010), Davis and Harrigan (2011), Amiti and Davis (2011), Harrigan and Reshef (2012), for particular focus on the distribution effects of trade integration.

uses one-factor one-good models to focus on the changes in the wage distribution within an industry whereas I propose a two-by-two-by-two model and study country industry differences in the skill premia. One other difference is that in most of the literature the results hinge on the existence of fixed exporting costs or, more generally, on the partition of firms between exporters and non exporters. The results in this paper instead do not rest on such partition.

In terms of the objectives the paper differs from the literature cited above since that literature studies the effect of trade integration on the skill premium while I study the effect of country-industry-firm characteristics on factor price (for any level of trade integration).

This is not the only paper to assume heterogeneity in factor intensity. Costinot and Vogel (2010) and Burstein and Vogel (2012) are notable examples. Their models differ from ours in terms of the market structure, technology, and preferences. The focuses are also very different; they (as well as Vannoorenberghe, 2012, who uses a model structure similar to ours) study the effect of trade liberalization on wage inequality, we study instead how countries and firm comparative advantage gives rise to differences in the skill premia across country and industries.

Reference to Helpman, Itskhoki and Redding (2010). This paper belongs to the class of models that use the standard Diamond-Mortensen-Pissarides approach to factor market frictions in trade models. After Helpman, Itskhoki and Redding (2010) a number of papers followed this approach: ..... What distinguishes my paper from this literature is a number of elements that I take on board. First, I allow factor proportions to influence search and matching costs. Not directly, but indirectly trhough factor prices. This is a natural extension to take on board since matching and screening are produced by factor inputs. Second, I take into account heterogeneity in factor intensity. I am not alone to do so. Indeed, Helpman, Itskhoki and Redding (2010) consider this case in one of the many research paths offered by their model (their Sect. 5.2) but they only study partial equilibrium results. As we shall see, when general equilibrium effects are taken into account, some results need to be revised. (see also technical appendix S5.4.) Crozet and Trionfetti (2013) consider heterogeneity in factor intensity but they assume perfectly competititve labour markets.

3 Stylised Facts

4 The Model Structure

The world economy is composed of two countries indexed by $c = A, B$; it produces two differentiated goods indexed by $i = Y, Z$, by using two heterogenous factors indexed by $j = H, L$ generically referred to as skilled and unskilled labour, respectively. Both factors are heterogeneous in terms of ability levels. Each country is endowed with a share $\vartheta_j^c > 0$ of world's endowments, $\overline{H}$ and $\overline{L}$. The technology of production, described below, is such that the skill intensity at the industry and firm level increases with an industry parameter ϕ_i. To fix ideas let Y be *H-intensive* and A be *H-abundant*: i.e., $\phi_Y > \phi_Z$ and $\vartheta_H^A \vartheta_L^B > \vartheta_H^B \vartheta_L^A$. International trade is subject to variable trade costs of the iceberg type by which for each unit shipped only a fraction $\tau \in [0, 1]$ arrives at destination.

Demand. The representative consumer's preferences is a Cobb-Douglas index with expenditure shares $\varepsilon_i \in (0, 1)$, $\varepsilon_Y + \varepsilon_Z = 1$ defined over CES aggregates of Y and Z whose elasticity of substitution between varieties is $\varsigma > 1$. The dual price index associated with each aggregate, denoted P_{ic}, is also a CES aggregate defined over the prices of all varieties of the same industry. Thus, the demand emanating from domestic residents and from foreign residents for the output of a firm in industry i of country A are, respectively:

$$q_{iAA} = (p_{iAA})^{-\varsigma} (P_{iA})^{\varsigma - 1} \varepsilon_i E_A; \qquad q_{iAB} = (p_{iAB})^{-\varsigma} (P_{iB})^{\varsigma - 1} \varepsilon_i E_B \tag{1}$$

where q is the quantity demanded and p is the price; the first and second country subscript tell us, respectively, where the variety is produced and where is consumed; total expenditure is denoted by E_c. Firms equalize marginal revenues between domestic and foreign market for any given total output. This allows writing sales as function of a demand shifter, denoted D_i^c, and total firm output q_i^c (see appendix Sect. 12.1):

$$s_i^A = \left(q_i^A\right)^{\frac{\varsigma-1}{\varsigma}} \underbrace{\left[\left(C_i^A\right)^{\varsigma} + \tau^{\varsigma-1}\left(C_i^B\right)^{\varsigma}\right]^{1/\varsigma}}_{D_i^A} \tag{2}$$

$$s_i^B = \left(q_i^B\right)^{\frac{\varsigma-1}{\varsigma}} \underbrace{\left[\left(C_i^B\right)^{\varsigma} + \tau^{\varsigma-1}\left(C_i^A\right)^{\varsigma}\right]^{1/\varsigma}}_{D_i^B} \tag{3}$$

Technology. Production requires continuously fixed and variable inputs. The variable input technology takes the CES form

$$q_i^c = \left[(1-\phi_i) \left\{ \bar{a}_L \left[l\alpha(t) \right]^{\gamma^c} \right\}^{\frac{\sigma-1}{\sigma}} + \phi_i \left\{ \bar{a}_H \left[h\beta(t) \right]^{\gamma^c} \right\}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where $\sigma > 1$ measures gross substitutability between factors, $\bar{a}_L$ and $\bar{a}_H$ denote firm-level average productivity of factors, $\phi_i \in (0,1)$ is the abevementioned industry parameter, and γ^c is a parameter contributing to total factor productivity which may differ between countries. For most of the paper I will drop the superscript to γ, I will resume it in Sect. **??** where I will discuss technology differences between countries. The variable t is a random variable with cumulative distribution $G(t)$ and with support (t_o, ∞) with $t_0 > 0$. Each firm draws randomly the value of t and remains associated with it until death of the firm will do them apart. At any point in time any firm has a probability of death equal to p_{death}. The continuous, non-decreasing, and differentiable functions $\alpha(t)$ and $\beta(t)$ contribute to determine factor productivity. Models that focus on Hicks-neutral heterogeneity assume $\alpha(t) = \beta(t)$ $\forall t$. I instead allow $\alpha(t) \neq \beta(t)$. Let $\flat(t) \equiv \beta(t)/\alpha(t)$. I will say that heterogeneity is *H-biased* if $\flat'(t) > 0$ $\forall t$; is *L-biased* if $\flat'(t) < 0$ $\forall t$; is *neutral* if $\flat'(t) = 0$ $\forall t$. Note that factor biased heterogeneity does not imply factor biased technology; indeed the (ex-ante) average values of $\alpha(t)$ and $\beta(t)$ may well be the same (no technology bias) and yet the heterogeneity may have a bias. This is the case when $\int_{t_0}^{\infty} \alpha(t)\, dG(t) = \int_{t_0}^{\infty} \beta(t)\, dG(t)$ and $\alpha'(t) \gtrless \beta'(t)$. Furthermore, we shall see that the direction of the bias is irrelevant for the results.

Firms face a fixed production cost, F_i, and a fixed entry cost, F_{ie}. Fixed production cost are paid for the set up of the firm and fixed entry costs are paid to draw t. Assuming homogeneous or heterogenous fixed costs gives qualitatively the same results. I assume homogenous fixed costs since this assumption allows focusing on heterogeneity in the production process, which is the heart of the matter. This is also the assumption most commonly retained in the literature (Melitz, 2003; Bernard, Redding and Schott, 2007; and many others). Specifically, I assume $F_i = \widetilde{mc}_i^c \digamma_i$ and $F_{ie} = \widetilde{mc}_i^c \digamma_{ie}$, where $\widetilde{mc}_i^c$ is the average marginal cost of production in industry i and country j while $\digamma_i$ and $\digamma_{ie}$ are scalars. This assumption represents the fixed cost as a quantity of output ($\digamma_i$, $\digamma_{ie}$) that must be produced by the firm and that ultimately cannot be sold. This interpretation is proposed in Yeaple (2005) and is widely used in the literature. Here, the unsalable output is produced by assembling all varieties of the industry-country output. Alternatively, but equivalently in terms of results, one could think of the fixed costs as the input of a homogenous composite good produced in a perfectly competitive market by assembling in a CES all the varieties of the industry output (similarly to Ethier, 1980).

5 Equilibrium of the Firm

Firms are profit maximizers. In addition to fixed and production costs they face matching and screening costs related to employment. A firm wanting to hire workers will first meet them (matching cost) and then verify the ability of workers to perform the job (screening costs). Matching and screening costs will be detailed in Sect. 6. Here I anticipate that to match with n_j workers the firm pays a cost equal to $b_j n_j$ and to identify workers with an ability level equal or higher than $\underline{a}_j$ the firm pays a screening cost equal to $c_j \left(\underline{a}_j\right)^{\delta} / \delta$, with $\delta > 0$. Thus, the profit of the firm is

$$\pi_i^c = s_i^c - \underbrace{\left[w_L\left(l\right)l + w_H\left(h\right)h\right]}_{Wage \quad Bill} - \underbrace{\left(b_L n_L + b_H n_H\right)}_{Search \quad Cost} - \underbrace{\frac{c_L\left(\underline{a}_L\right)^{\delta} + c_H\left(\underline{a}_H\right)^{\delta}}{\delta}}_{Screening \quad Cost} - F_i \quad (5)$$

Firms optimize over employment, l and h, over the number of workers to match with, n_L and n_H, and over the threshold ability levels, $\underline{a}_L$ and $\underline{a}_H$. Furthermore, firm-level wage negotiation gives endogenously the wages functions, $w_L\left(l\right)$ and $w_H\left(h\right)$. For clarity of exposition I will mark by a $^\circ$ firm-level optimal values of wages, matching, and screening. I will drop the country and industry superscript on these variables but it should remain clear that they depend on country characteristics through general equilibrium and on industry characteristics through sectoral and general equilibrium.

5.1 Wage Determination

Matching and screening costs are sunk when wages are negotiated. The firm and the workers engage in a bargaining game with equal weights over wages in the way proposed by Stole and Zwiebel (1996a, 1996b). The solution of the game is a function for each factor (the wage function) that satisfy the requirement that the marginal benefit for a firm from highiring a worker is equal to the worker's benefit from accepting the job. With income from giving up the job normalized to zero, the benefit from accepting the job is exactly the salary. Given that the only information revealed by screening is whether a worker skill is above or below $\underline{a}_j$, neither the frm nor the workers can observe the match-specific skill of a worker. Each worker is therefore treated as if his skill level were equal to the average level $\overline{a}_j$. This, makes that the bargeining game takes place under symmetric information. Thus, the functions that satisfy the Stole-Zwiebel condition must obay the following differential equations where the worker's marginal contribution to profits equals the workers wage:

$$\underbrace{\frac{ds_{id}^c}{dl}-\frac{dw_L}{dl}l-w_L}_{\text{marg. contr. to profit}} = w_L \quad \Rightarrow \overset{\circ}{w}_L(l)=\frac{D_i^c\left[\bar{a}_L(l\alpha)^{\gamma}\right]^{\frac{\sigma-1}{\sigma}}\gamma(1-\phi)}{l\left(\gamma+\frac{\sigma}{\sigma-1}\right)} \tag{6}$$

$$\underbrace{\frac{ds_{id}^c}{dh}-\frac{dw_H}{dh}h-w_H}_{\text{marg. contr. to profit}} = w_H \quad \Rightarrow \overset{\circ}{w}_H(h)=\frac{D_i^c\left[\bar{a}_H(h\beta)^{\gamma}\right]^{\frac{\sigma-1}{\sigma}}\gamma\phi}{h\left(\gamma+\frac{\sigma}{\sigma-1}\right)} \tag{7}$$

Using the solutions of the two differential equations reported on the rigth of (6)-(7) we obtain the firm-level wage bill and the profit as a constant fraction of sales:[2]

$$\overset{\circ}{w}_L(l)\,l+\overset{\circ}{w}_H(h)\,h = \frac{\gamma(\sigma-1)}{\gamma(\sigma-1)+\sigma}s_i^c, \tag{8}$$

$$\pi_i^c = \frac{\sigma}{\gamma(\sigma-1)+\sigma}s_i^c-F_i. \tag{9}$$

5.2 Employment Determination

Firm employment depends on the number of matches and on the severity of screening. Thus, optimizing over employment is tantamount to optimizing over matching and screening. Let the distribution of ability be Pareto with shape parameter $\chi>1$ and lower bounds $l_0=h_0=1$ for unskilled and skilled labour, respectively.[3] Then employment and average productivity are

$$l = n_L\left(\frac{l_0}{\underline{a}_L}\right)^{\chi}, \qquad h=n_H\left(\frac{h_0}{\underline{a}_H}\right)^{\chi}, \tag{10}$$

$$\bar{a}_L = \frac{\chi\underline{a}_L}{\chi-1}, \qquad \bar{a}_H=\frac{\chi\underline{a}_H}{\chi-1}. \tag{11}$$

[2]The properties of the wage functions $\overset{\circ}{w}_j$ may be obtained for any σ and ς but these functions may be written explicitly only when we assume $\sigma=\varsigma$. I will use this assumption in the analytical results.

[3]We could assume different shape parameters as well as different lower bounds between factors but, remember, our objective is to study the relationship between the comparative advantage of couintries and the skill premium. We therefore want to stay as close as possible to the Heckscher-Ohlin model. Indeed, the only modification to the HO model is to introduce heterogeneity in relative marginal productivity of factors.

When deciding employment the firm cannot optimize over wages since the only information the firm has is how wages are determined. The firm anticipates the wage functions but knows neither the ability of each individual worker nor the average ability of the workers with whom it will be matched. Thus, at the stage of employment determination the firm maximizes profits over n_L, n_H, $\underline{a}_L$, $\underline{a}_H$ given the wage functions (6)-(7). The first order conditions for profit maximization give $\overset{\circ}{n}_L$, $\overset{\circ}{n}_H$, $\overset{\circ}{\underline{a}}_L$, $\overset{\circ}{\underline{a}}_H$. Replacing these optimal values into (10)-(11) give optimal employment $\overset{\circ}{l}$ and $\overset{\circ}{h}$ and, finally, trhough (6) and (7) we obtain $\overset{\circ}{w}_L$ and $\overset{\circ}{w}_H$. Positiveness of $\overset{\circ}{n}_L$, $\overset{\circ}{n}_H$ is assured by assuming $0 < \gamma < 1/\chi < 1$ (see appendix Sect. 12.2 for the employment determination and parametric restrictions).

6 Matching and Screening

I follow the Diamond-Mortensen-Pissarides approach. Matching is the result of a random process. The total number of matches, N_j, is a Cobb-Douglas function of the total number of vacancies, V_j, and of the mass of workers looking for a job:

$$N_L = \mu_1 V_L^{\mu_2} \underline{L}^{1-\mu_2}, \qquad N_H = \mu_1 V_H^{\mu_2} \underline{H}^{1-\mu_2}, \tag{12}$$

with $0 < \mu_1, \mu_2 < 1$. Let v_j denote the number of vacancies posted by a firm and assume that the number of matches accruing to a firm, n_j, is proportional to the firm's share in total vacancies: $n_L = (v_L/V_L) N_L$, and $n_H = (v_H/V_H) N_H$. Then, a firm wanting to match with n_j workers needs to post $v_j > n_j$ vacancies equal to $v_j = V_j n_j / N_j$, which, using the matching technology (12), becomes $v_j = (1/\mu_1)^{1/\mu_2} (x_j)^{(1-\mu_2)/\mu_2} n_j$. Let me assume for the moment that the matching activity is performed by matching agency operating in perfect competition both in the labor market and in the market for matching services. Being perfectly competitive, matching firm price at marginal cost and pay factors at the ex-ante expected wages w_{Le}^c and w_{He}^c. Let us assume that posting a vacancy for factor j is an *H-intensive* activity and, to simplify matters, assume that posting a vacancy for factor j requires one unit of factor j only. Then, the per-worker search cost, b_j^c, is equal to

$$b_j^c = w_{je}^c \left(\frac{1}{\mu_1}\right)^{\frac{1}{\mu_2}} (x_j)^{\frac{1-\mu_2}{\mu_2}} \tag{13}$$

Given free entry and exit in the labor market, the ex-ante expected wages (w_{Le}^c, w_{He}^c) must equal the outside options (w_{Lo}^c, w_{Ho}^c). To determine the expected wages we use the first order conditions that lead to employment determination. In appendix Sect. (12.2) [DA FARE] I show that the expected wage conditional on matching with a specific firm ($\overset{\circ}{w}_L l^\circ/\overset{\circ}{n}_L$ or $\overset{\circ}{w}_H h^\circ/\overset{\circ}{n}_H$) is equal to the unit search cost, that is: $\overset{\circ}{w}_L l^\circ/\overset{\circ}{n}_L = b_L$ and $\overset{\circ}{w}_H h^\circ/\overset{\circ}{n}_H = b_H$. Let N_j be the total number of matches occurring for factor j and let $\underline{L}$ and $\underline{H}$ be the number of $L-$ and $H-$*workers* looking for a job. The probability of being matched with some firm is, respectively, $N_L/\underline{L}$ and $N_H/\underline{H}$. The expected wage conditional on being matched with some firm is equal to the expected wage conditional on matching with a firm times the probability of being matched with some firm. That is:

$$w^c_{Le} = \overset{\circ}{w}_L \frac{l^\circ}{\overset{\circ}{n}_L} \frac{N_L}{\underline{L}} = x_L b_L, \quad w^c_{He} = \overset{\circ}{w}_H \frac{h^\circ}{\overset{\circ}{n}_H} \frac{N_H}{\underline{H}} = x_H b_H \tag{14}$$

where the second subscript $_e$ denotes the expected value of the variable while $x_L \equiv N_L/L$ and $x_H \equiv N_H/H$ denote the market tightness.

Using (14) the arbitrage conditions that ex-ante expected wages must equal the outside option are

$$w^c_{jo} = \underbrace{x_j b_j}_{w^c_{je}} \tag{15}$$

Equations (13) and (15) may be solved to yield $b^c_j = w^c_{jo}/\mu_1$, and $x_j = \mu_1$ for $j = H, L$. As a result, the relative cost of matching with factor H, given by

$$\frac{b^c_H}{b^c_L} = \frac{w^c_{Ho}}{w^c_{Lo}} \equiv \omega^c_o, \tag{16}$$

is lower in the *H-abundant* country as long as ω^c_o is lower in this country.

It is important to note that this result is not the consequence of the assumption that providing matching services for factor j is a j intensive activity. This result is due to the arbitrage conditions (15). Indeed, as often done in the literature, we could insert an outside good in the model (taken as numéraire) and assume that the cost of posting a vacancy is equal to μ_0 units of such good. Then, we would have $b^c_H/b^c_L = (\omega^c_o)^{1-\mu_2}$. The result remains that the relative cost of matching is lower in the *H-abundant* as long as ω^c_o is lower in this country.

Coming to screening we assume that screening factor j is a *j-intensive* activity (one needs economists to screen economists). Screening is performed by perfectly competitive screening firms that pay ex ante expected wages. Specifically, we assume that c_j is one unit of factor j, therefore $c_j = w^c_{jo}$. The relative cost of screening for factor h, given by

$$\omega^c_o \left(\frac{\overset{\circ}{\underline{a}}_H}{\overset{\circ}{\underline{a}}_L} \right)^\delta , \tag{17}$$

is, ceteris paribus, lower in the *H-abundant* country as long as ω_o^c is lower in this country.[4]

One last comment is in order. One may alternatively assume that sampling and screening takes place within the firm and that factors employed in these activity are paid firm-level wages. This would leave the main result unchanged because of the general equilibrium effect of comparative advantage on relative cost of matching and screening [DA VERIFICARE]. I will come back to this matter in Sect. 10.

7 Ranking of cut off values.

The set up of general equilibrium equations for this class of models is well known and we therefore detail it in appendix Sect. 12.3. Here we focus on the ranking of cut off values emerging from general equilibrium. Such ranking allows comparing average skill premia across countries and industries. The proof of cut off ranking is left in appendix Sect. 12.4 but it is instructive to discuss here its economic logic. Consider first the case of *H-biased* heterogeneity and consider two firms belonging to different industries and countries but with identical draw t' larger than the cut off value. The sales and profit of the cut off firm is larger in the skill intensive industry and in the skill abundant country. The reason is that both firms use H more intensively than the cut off firm but H is more productive in the *H-intensive* industry and cheaper in the *H-abundant* country. Thus the firm in the *H-intensive* industry and *H-abundant* country (refer to it as *HH* situation) is better off than the firm in the *LL* situation, ceteris paribus. This means that, for any t' the profitability is higher for *HH* firms than for *LL* firms. But ex-ante expected profits must be the same (free entry condition) and this is possible only if firm selection is tougher in the *H-intensive* industry and in the *H-abundant* country. Thus, $t_Y^{*c} > t_Z^{*c}$ and $t_i^{*A} > t_i^{*B}$. If we assumed *L-bised* heterogeneity we would have the opposite, namely: $t_Y^{*c} < t_Z^{*c}$ and $t_i^{*A} < t_i^{*B}$. Lastly, if heterogeneity is neutral then $t_Y^{*c} = t_Z^{*c}$ and $t_i^{*A} = t_i^{*B}$. With this in mind we may now rank the average values of $\flat(t)$.

Let $\widetilde{\flat}_i^c = \left[\frac{1}{1-G\left(t_i^{*c}\right)}\int_{t_i^{*c}}^{\infty}\left[\flat(t)\right]^{\frac{-\chi\gamma_s}{\Delta}} g(t)\,dG\right]^{-\frac{\Delta}{\chi\gamma_s}}$ be an average of $\flat(t)$ and

[4] Again, alternative assumptions are equally plausible. In particular, it may be assumed that screening is performed by the personnel department of the firm. Given that the personnel department employs non-production workers, the wage paid to these workers is the ex-ante expected wage.

note that $d\widetilde{\flat}_i^c/dt_i^{*c} \gtreqless 0 \Leftrightarrow \flat'(t) \gtreqless 0$. That is, $\widetilde{\flat}_i^c$ is increasing or decreasing in dt_i^{*c} depending on whether heterogeneity is *H-biased* or *L-biased*. The ranking of t_i^{*c} and the sign of $d\widetilde{\flat}_i^c/dt_i^{*c}$ allow ranking $\widetilde{\flat}_i^c$ by pair as follows (see Sect. 12.4):

$$\widetilde{\flat}_i^A/\widetilde{\flat}_i^B > 1, \quad \widetilde{\flat}_Y^c/\widetilde{\flat}_Z^c > 1 \text{ for } \flat'(t) \neq 0 \tag{18}$$

$$\widetilde{\flat}_i^A/\widetilde{\flat}_i^B = \widetilde{\flat}_Y^c/\widetilde{\flat}_Z^c = 1 \quad \text{for} \quad \flat'(t) = 0 \tag{19}$$

This ranking is used in Sect. 10 below.

8 Comparative firm-level skill premia

I compare skill premia between firms belonging to different countries and industries. Using (6)-(7) to write the skill premium, $\omega_i^c \equiv \overset{\circ}{w}_H/\overset{\circ}{w}_L$, and replacing $\overset{\circ}{n}_L$, $\overset{\circ}{n}_H$, $\overset{\circ}{\underline{a}}_L$, $\overset{\circ}{\underline{a}}_H$ in this expression we obtain

$$\omega_i^c = (\Phi_i)^{-\frac{\chi}{\Delta}} (\omega_o^c)^{\frac{(\mathfrak{s}\gamma-1)(\delta-\chi)+\mathfrak{s}}{\Delta}} [\flat(t)]^{\frac{-\chi\gamma\mathfrak{s}}{\Delta}} \tag{20}$$

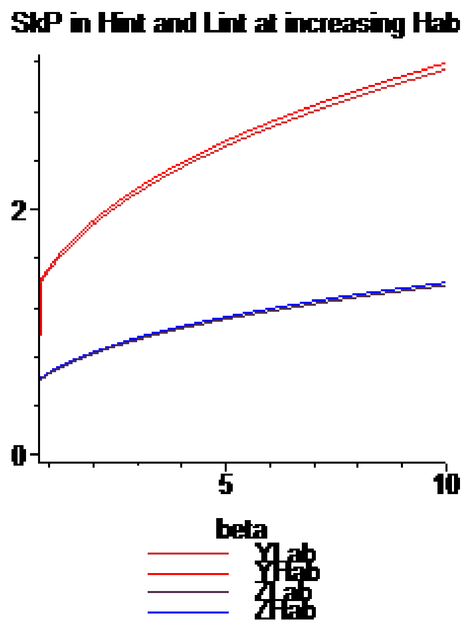
where $0 < \mathfrak{s} = \frac{\sigma-1}{\sigma} < 1$, $\Phi_i = \phi_i/(1-\phi_i)$ and $\Delta \equiv \gamma\mathfrak{s}(\delta-\chi) + \mathfrak{s} - \delta < 0$.[5] We see from expression (20) that the skill premium increases with the relative marginal productivity of H proxied by $\flat(t) \equiv \beta(t)/\alpha(t)$. More interestingly, the skill premium exhibits supermodularity in $(\Phi_i, \flat(t))$. Thus, firms with higher relative marginal productivity of H pay higher skill primia and more so if they belong to the skill intensive industry. The outside option, ω_o^c, may relate positively or negatively to the skill premium depending on $(\mathfrak{s}\gamma-1)(\delta-\chi)+\mathfrak{s} \lesseqgtr 0$. When this inequality holds as $(\mathfrak{s}\gamma-1)(\delta-\chi)+\mathfrak{s} < 0$ firms with higher relative marginal productivity of H pay higher skill premia and more so if they belong to the *H-abundant* country (low ω_o^c). Analogously, mutatis mutandi, when the inequality holds as $(\mathfrak{s}\gamma-1)(\delta-\chi)+\mathfrak{s} > 0$. Visually, these results are summarized in Fig 1.

LEGEND: YLab = skill-intensive sector of the Labor-abundant country, YHab = skill-intensive sector of the Skill-abundant country, etc.

We may summarize these results in the following proposition.

Proposition 1 *Firm level skill premia depends on firm, country, and industry characteristics. Furthermore, there is supermodularity between firm-level and industry skill intensity; and there is super or sub modularity between firm-level skill intensity and country skill-abundance.*

[5] L'espressione 12.2 è praticamente identica a HIR eq (S32). Ma loro si fermano al partial equilibrium e a b_H/b_L.



These results are almost those shown in Figures ?? and ??. The difference is that the regressions represented in the figures used firm-level average wage instead of firm-level skill premium. I therefore need to discuss the firm-level average wage.

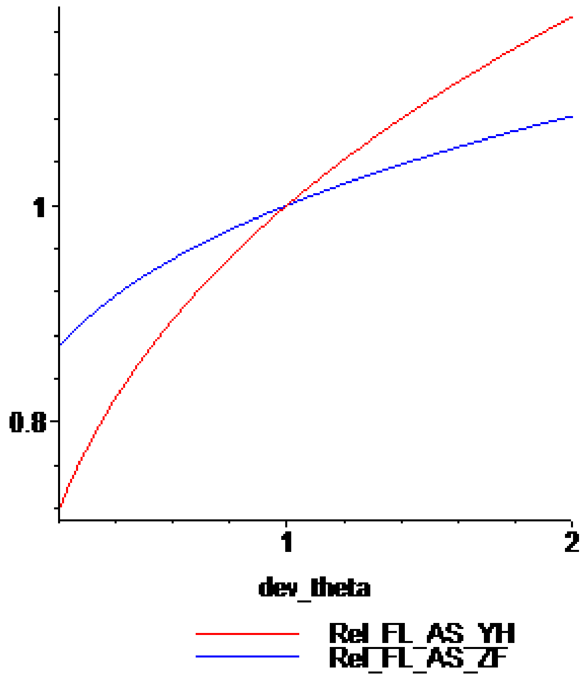
9 Comparative firm-level average wage

Firm-level average wage is defined as

$$\varpi_i^c(t) \equiv \frac{w_L^\circ l^\circ + w_H^\circ h^\circ}{l^\circ + h^\circ} \tag{21}$$

The average wage, ϖ_i^c, bears a country and industry index since all the firm-level optimal variables $^\circ$ depend on country industry characteristics in addition to the firm-level random draw. The model gives a precise prediction in terms of relative average wage,$\hat{w}_i^c(\varrho) \equiv \varpi_i^c\left(\varrho\widetilde{\flat}_i^c\right)/\varpi_i^c\left(\widetilde{\flat}_i^c\right)$ where $\varrho > 0$. In appendix Sect. (**??**) I show ....

So two firms with identical ϱ but belonging to different industries and



countries will pay a different relative average wage in this order:

$$\hat{w}_i^H(\varrho) \gtreqless \hat{w}_i^F(\varrho) \text{ as } \varrho \gtreqless 1 \tag{22}$$

$$\hat{w}_Y^c(\varrho) \gtreqless \hat{w}_Z^c(\varrho) \text{ as } \varrho \gtreqless 1 \tag{23}$$

Graphically, this means
LEGEND: Rel_FL_AS_YH $= \hat{w}_Y^H(\varrho)$, Rel_FL_AS_ZF $= \hat{w}_Z^F(\varrho)$.
This figure is exactly as in the stylised facts.

10 Comparative average skill premia

I now compare average skill premia across country and industries. Using expression (20) and the ranking $\widetilde{\flat}_i^c$ in expressions (18)-(19) we may establish a number of results.

The first result concerns the comparison of average skill premia and obteins from the following expression:

$$\frac{\overline{\omega}_Y^A\left(t_Y^{*A}\right)}{\overline{\omega}_Z^B\left(t_Z^{*B}\right)} = \left(\frac{\Phi_Y}{\Phi_Z}\right)^{-\frac{\chi}{\Delta}} \left(\frac{\omega_o^A}{\omega_o^B}\right)^{\frac{(s\gamma-1)(\delta-\chi)+s}{\Delta}} \left(\frac{\widetilde{\flat}_Y^A}{\widetilde{\flat}_Z^B}\right)^{\frac{-\chi\gamma s}{\Delta}} \tag{24}$$

Each of the three multiplicands in parenthesis represents one of the mechanism of the model. First, ceteris paribus, the average skill premium is higher in the skill intensive industry because of higher Φ_Y/Φ_Z. The reason is that workers are paid the marginal contribution to profit and such contribution exhibits supermodularity in (Φ_Y, h). This is intuitive but to grasp the math of it remember that profits depend on sales which depend on output (see expression 2 or 3) and that output exibits supermodularity in (Φ_Y, h), see (4). Second, consider the role ω_o^A/ω_o^B. Factor proportions make $\omega_o^A/\omega_o^B > 1$ which may make the average skill premium in the skill abundant country higher than in the skill scarce country, ceteris paribus, depending on the sign of $[(\mathfrak{s}\gamma - 1)(\delta - \chi) + \mathfrak{s}]/\Delta$. It is worth examining this mechanism in greater detail. To this end consider a capital abundant country, i.e., low ω_o^c. Low ω_o^c makes the relative cost of *H-matching* and *H-screening* low and induces firms to sample relatively more H and to increase the relative severity of screening H. More *H-matching* pushes *H-intensity* up via expressions (10) which depresses the skill premium. More severe *H-screening* has two effects: pushes the relative average productivity of H up (see expressions 11) and reduces the *H-intensity* (see expression 10); both of these effects push the skill premium up. If the two screening effects are stronger than the matching effect, then the skill premium is higher in the skill abundant country, ceteris paribus. Third, non neutral heterogeneity results in $\widetilde{\flat}_Y^A/\widetilde{\flat}_Z^B > 1$ thereby making the average skill premium higher in the skill-intensive industry and in the skill abundant country, ceteris paribus.

The results above may be summarized by the following proposition:

Proposition 2 *The (average) relative price of a factor may relate positively or negatively to its relative abundance.*

This result is simple, robust, new, and coherent with factor proportion theories. Simple, because it stems quite intuitively from a simple model. Robust, because a number of different assumptions about matching and screening technologies may be adopted without affecting the results. New because it runs against the traditional mapping between relative factor price and factor proportions where relative abundance unambiguously reduces the relative price of a factor (in costly trade). It is coherent with the theory of factor proportions, however, because the expected factor price (or, which is the same, outside options) relate to factor proportions in the traditional "Heckscher-Ohlin" way. Indeed, it is precisely because of the traditional effect of factor proportions on expected factor price that realized factor price may run against the traditional mapping.

It is now time to go back to my last comment in Sect. 6 where I allluded to a model variant where matching and screening is performed internally by the firm at firm's wages. To obtain the results of this variance just replace ω_o^c with $\omega_i^c(t)$ in all the expressions above in this section. We see the average skill premium would be *certainly* higher in the skill intensive industry and in the skill abundant country. Specifically, expression (24) becomes:

$$\frac{\overline{\omega}_Y^A\left(t_i^{*A}\right)}{\overline{\omega}_Z^B\left(t_i^{*B}\right)} = \left(\frac{\Phi_Y}{\Phi_Z}\right)\left(\frac{\widetilde{\flat}_Y^A}{\widetilde{\flat}_Z^B}\right)^{-\gamma\varsigma} > 1. \tag{25}$$

11 Conclusion

To be written.

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12 Appendix

This appendix provides the main mathematical passages used to obtain the results in the text.

12.1 Equalization of marginal revenues

Firms equalize marginal revenues in the domestic and foreign market. Inverting the demand functions (1) and taking into account that to send one unit abroad the firm must produce $1/\tau$ units allows writing sales of a firm in A as

$$s_i^A = (q_{iAA})^{\frac{\varsigma-1}{\varsigma}} \overbrace{(P_{iA})^{\frac{\varsigma-1}{\varsigma}} (\varepsilon_i E_A)^{1/\varsigma}}^{C_i^A} + \tau^{\frac{\varsigma-1}{\varsigma}} \left(q_{iAB}^A\right)^{\frac{\varsigma-1}{\varsigma}} \overbrace{(P_{iB})^{\frac{\varsigma-1}{\varsigma}} (\varepsilon_i E_B)^{1/\varsigma}}^{C_i^B} \quad (26)$$

where q_{iAB} is the quantity demanded in B of a variety produced in A and q_{iAB}^A is the quantity produced in A such that the quantity q_{iAB} arrives at destination, i.e., $q_{iAB} = \tau q_{iAB}^A$. The horizontal parenthesis $\overbrace{\quad}$ define C_i^c. Analogous expressions apply to s_i^B and its components. Using (26), marginal sales are

$$\frac{ds_i^A}{dq_{iAA}} = \frac{\varsigma - 1}{\varsigma} (q_{iAA})^{\frac{-1}{\varsigma}} C_i^A; \quad \text{and} \quad \frac{ds_i^A}{dq_{iAB}^A} = \tau^{\frac{\varsigma-1}{\varsigma}} \frac{\varsigma - 1}{\varsigma} \left(q_{iAB}^A\right)^{\frac{-1}{\varsigma}} C_i^B. \quad (27)$$

Equalizing the marginal sale between markets we obtain

$$q_{iAB}^A = \tau^{\varsigma-1} \left(\frac{C_i^B}{C_i^A}\right)^{\varsigma} q_{iAA}. \quad (28)$$

Replacing (28) into (26) gives

$$s_i^A = (q_{iAA})^{\frac{\varsigma-1}{\varsigma}} C_i^A \left[1 + \tau^{\varsigma-1} \left(\frac{C_i^B}{C_i^A}\right)^{\varsigma}\right] \quad (29)$$

and, using the expression for total firm output, $q_i^A = q_{iAA} + q_{iAB}^A$ to replace q_{iAA} into (29) we obtain expression (2) in the main text. Analogously we obtain expression (3).

12.2 Employment determination, factor intensity, skill premium.

The details of the mathematical passages are in the Maple file associated to this paper and available at www.trionfetti.wordpress.com . We report here the main mathematical passages for convenience of the reader. The four FOC for profit maximization are

$$\left.\frac{d\pi_i^c}{dn_j}\right|_{w_L^\circ, w_H^\circ} = 0, \qquad \left.\frac{d\pi_i^c}{d\underline{a}_j}\right|_{w_L^\circ, w_H^\circ} = 0, \quad i = Y, Z, \quad j = L, H. \tag{30}$$

which may be written as

$$\begin{aligned}
A_{iL}^c \left(\underline{a}_L\right)^{\mathfrak{s}(1-\gamma\chi)} \left(n_L\right)^{\gamma\mathfrak{s}-1} (\alpha)^{\gamma\mathfrak{s}} &= b_L & (31)\\
A_{iH}^c \left(\underline{a}_H\right)^{\mathfrak{s}(1-\gamma\chi)} \left(n_H\right)^{\gamma\mathfrak{s}-1} (\beta)^{\gamma\mathfrak{s}} &= b_H & (32)\\
B_{iL}^c \left(n_L\right)^{\gamma\mathfrak{s}} \left(\underline{a}_L\right)^{\mathfrak{s}(1-\chi\gamma)-\delta} \alpha^{\gamma\mathfrak{s}+\xi} &= -ß & (33)\\
B_{iH}^c \left(n_H\right)^{\gamma\mathfrak{s}} \left(\underline{a}_H\right)^{\mathfrak{s}(1-\chi\gamma)-\delta} \beta^{\gamma\mathfrak{s}+\xi} &= -ß & (34)
\end{aligned}$$

where $\mathfrak{s} = \frac{\sigma-1}{\sigma} \in (0,1)$ and

$$\begin{aligned}
A_{iH}^c &= \left(\frac{\chi}{1-\chi}\right)^{\mathfrak{s}} \frac{\mathfrak{s}\gamma\phi}{1+\mathfrak{s}\gamma} D_i^c, & (35)\\
A_{iL}^c &= \left(\frac{\chi}{1-\chi}\right)^{\mathfrak{s}} \frac{\mathfrak{s}\gamma(1-\phi)}{1+\mathfrak{s}\gamma} D_i^c, & (36)\\
B_{iL}^c &= \frac{(1-\phi)\left[\left(\chi^{\mathfrak{s}-1}-\chi^{\mathfrak{s}}\right)\mathfrak{s} + \left(\chi^{\mathfrak{s}+1}-\chi^{\mathfrak{s}}\right)\mathfrak{s}\gamma\right]\chi}{(1+\mathfrak{s}\gamma)(1-\chi)(\chi-1)^{\mathfrak{s}}} D_i^c, & (37)\\
B_{iH}^c &= \frac{\phi\left[\left(\chi^{\mathfrak{s}-1}-\chi^{\mathfrak{s}}\right)\mathfrak{s} + \left(\chi^{\mathfrak{s}+1}-\chi^{\mathfrak{s}}\right)\mathfrak{s}\gamma\right]\chi}{(1+\mathfrak{s}\gamma)(1-\chi)(\chi-1)^{\mathfrak{s}}} D_i^c, & (38)\\
ß &= \frac{\chi\left(\chi^{\delta}-\chi^{\delta-1}\right)(1+\gamma\mathfrak{s})}{(1+\mathfrak{s}\gamma)(1-\chi)} \left(\frac{1}{\chi-1}\right)^{\delta}, & (39)
\end{aligned}$$

FOCs (32)-(34) may be solved explicitly for the four endogenous variables $n_j^\circ, \underline{a}_j^\circ$. Naturally, we impose restrictions on parameters such that it is optimal for the firm to sample some workers, $n_j^\circ > 0$, otherwise employment would be zero and production would not take place. Whenever sampling is positive, no matter how severe screening is, employment is positive and production takes place. The condition for positive sampling is $(1-\gamma\chi)(\chi-1) > 0$ which we satisfy by assuming $0 < \gamma < 1/\chi < 1$ (remember that $\chi > 1$ is required by finiteness of the average of the Pareto distribution). This restrictions also imply $\gamma\mathfrak{s} < 1$, $\gamma\chi < 1$, and $\Delta \equiv \gamma\mathfrak{s}(\delta-\chi) + \mathfrak{s} - \delta < 0$ which will be used below. Dividing (32) by (31) and (34) by (33) and then rearranging we obtain the explicit solutions for the four ratios of interest:

$$\frac{n_H^{\circ}}{n_L^{\circ}} \equiv \eta_i^c = (\Phi_i)^{\eta_1}\left(\frac{b_H}{b_L}\right)^{\eta_2}\left(\frac{c_H}{c_L}\right)^{\eta_3}[\flat(t)]^{\eta_4} \tag{40}$$

$$\frac{a_H^{\circ}}{a_H^{\circ}} \equiv \psi_i^c = (\Phi_i)^{\psi_1}\left(\frac{b_H}{b_L}\right)^{\psi_2}\left(\frac{c_H}{c_L}\right)^{\psi_3}[\flat(t)]^{\psi_4} \tag{41}$$

$$\frac{h^{\circ}}{l^{\circ}} \equiv \theta_i^c = (\Phi_i)^{\theta_1}\left(\frac{b_H}{b_L}\right)^{\theta_2}\left(\frac{c_H}{c_L}\right)^{\theta_3}[\flat(t)]^{\theta_4} \tag{42}$$

$$\frac{w_H^{\circ}}{w_L^{\circ}} \equiv \omega_i^c = (\Phi_i)^{\omega_1}\left(\frac{b_H}{b_L}\right)^{\omega_2}\left(\frac{c_H}{c_L}\right)^{\omega_3}[\flat(t)]^{\omega_4} \tag{43}$$

where:

$$\eta_1 = -\frac{\delta}{\Delta} > 0; \quad \eta_2 = \frac{\delta - \mathfrak{s}(1-\gamma\chi)}{\Delta} < 0\ ; \quad \eta_3 = \frac{\mathfrak{s}(1-\gamma\chi)}{\Delta} < 0 \tag{44}$$

$$\eta_4 = \frac{-\gamma\delta\mathfrak{s}}{\Delta} > 0; \quad \psi_1 = -\frac{1}{\Delta} > 0\ ; \quad \psi_2 = \frac{\gamma\mathfrak{s}}{\Delta} < 0 \tag{45}$$

$$\psi_3 = \frac{1-\gamma\mathfrak{s}}{\Delta} < 0\ ; \quad \psi_4 = \frac{-\gamma\mathfrak{s}}{\Delta} > 0\ ; \quad \theta_1 = -\frac{\delta-\chi}{\Delta} > 0; \tag{46}$$

$$\theta_2 = \frac{\delta-\mathfrak{s}}{\Delta} < 0\ ; \quad \theta_3 = \frac{\mathfrak{s}-\chi}{\Delta} > 0\ ; \quad \theta_4 = -\frac{\gamma\mathfrak{s}(\delta-\chi)}{\Delta} > 0 \tag{47}$$

$$\omega_1 = -\frac{\chi}{\Delta} > 0\ ; \quad \omega_2 = \frac{\delta\gamma\mathfrak{s}+\mathfrak{s}-\delta}{\Delta} \lesseqgtr 0 \Leftrightarrow \delta \gtreqless \frac{\mathfrak{s}}{1-\gamma\mathfrak{s}} \tag{48}$$

$$\omega_3 = -\frac{(\gamma\mathfrak{s}-1)\chi}{\Delta} < 0\ ; \quad \omega_4 = -\frac{\chi\gamma\mathfrak{s}}{\Delta} > 0 \tag{49}$$

Substituting ω_o in relative sampling and screening we obtain expression (20) in the text.

12.3 General Equilibrium

To go from firm equilibrium to general equilibrium we have to go through three steps: sectorial equilibrium, aggregation, general equilibrium.

Sectorial Equilibrium. Using (2) or (3) it is apparent that the sales ratio for two firms in the same industry and country depends only on relative output which, ultimately, depends only on the values of t drawn by the firms. Thus, for any t' and t'':

$$\frac{s_i^c(t')}{s_i^c(t'')} = \left[\frac{q_i^c(t')}{q_i^c(t'')}\right]^{\frac{\varsigma-1}{\varsigma}} \tag{50}$$

Let t_i^{*c} denote the cut off value of t in industry i of country c such that profit is zero; $\pi_i^c\left(t_i^{c*}\right)=0$. Using this zero cut off profit condition we obtain the sales of the cut off firms

$$s_i^c\left(t_i^{c*}\right)=\left[\gamma\left(\sigma-1\right)+\sigma\right]F_i \tag{51}$$

Using (51) into (50) we obtain the sales of any firm as function of t and t_i^{c*}:

$$s_i^c\left(t\right)=\left[\frac{q_i^c\left(t\right)}{q_i^c\left(t_i^{c*}\right)}\right]^{\frac{\varsigma-1}{\varsigma}}\left[\gamma\left(\sigma-1\right)+\sigma\right]F_i. \tag{52}$$

At this point of the analysis, all firm variables depend only on cut off values which will be determined in general equilibrium.

Aggregation. To write the general equilibrium set of equations we need aggregate individual sales, profit, and factor demands.

Average sales and profit are:

$$\overline{s}_i^c\left(t_i^{*c}\right) = \frac{\left[\gamma\left(\sigma-1\right)+\sigma\right]F_i}{1-G\left(t_i^{*c}\right)}\int_{t_i^{*c}}^{\infty}\left[\frac{q_i^c\left(t\right)}{q_i^c\left(t_i^{*c}\right)}\right]^{\frac{\varsigma-1}{\varsigma}}dG \tag{53}$$

$$\overline{\pi}_i^c\left(t_i^{*c}\right) = \frac{\overline{s}_i^c}{\gamma\left(\sigma-1\right)+\sigma}-F_i. \tag{54}$$

Note that $\overline{s}_i^c$ and $\overline{\pi}_i^c$ are also the expected sale and profit of a firm prior to entry.

Average factor demand in production is

$$\overline{l}_{i,pr}^c\left(t_i^{*c}\right)=\int_{t_i^{*c}}^{\infty}\frac{\overset{\circ}{l}_i\left(t\right)}{1-G\left(t_i^{*c}\right)}dG,\qquad \overline{h}_{i,pr}^c\left(t_i^{*c}\right)=\int_{t_i^{*c}}^{\infty}\frac{\overset{\circ}{h}_i\left(t\right)dG}{1-G\left(t_i^{*c}\right)}. \tag{55}$$

average demand for fixed inputs takes the same functional forms as (55) except that the scalar F replaces the demand shifter. Average factor demand for posting vacancies is

$$\overline{l}_{i,v}^c\left(t_i^{*c}\right)=\int_{t_i^{*c}}^{\infty}\frac{\overset{\circ}{n}_L\left(t\right)}{\left[1-G\left(t_i^{*c}\right)\right]}dG,\ \ \overline{h}_{i,scr}^c\left(t_i^{*c}\right)=\int_{t_i^{*c}}^{\infty}\frac{\overset{\circ}{n}_H\left(t\right)}{\left[1-G\left(t_i^{*c}\right)\right]}dG \tag{56}$$

Average factor demand for screening is

$$\overline{l}_{i,scr}^c\left(t_i^{*c}\right)=\int_{t_i^{*c}}^{\infty}\frac{\left[\overset{\circ}{a}_L\left(t\right)\right]^{\delta}}{\left[1-G\left(t_i^{*c}\right)\right]\delta}dG,\ \ \overline{h}_{i,scr}^c\left(t_i^{*c}\right)=\int_{t_i^{*c}}^{\infty}\frac{\left[\overset{\circ}{a}_H\left(t\right)\right]^{\delta}}{\left[1-G\left(t_i^{*c}\right)\right]\delta}dG \tag{57}$$

Total average factor demands are $\overline{l}_i^c\left(t_i^{*c}\right)=\sum_d\overline{l}_{i,d}^c\left(t_i^{*c}\right)$ where $d=pr,v,scr$.

General Equilibrium. There are four sets of equilibrium equations that apply to every country and industry.

First, stationarity of the equilibrium requires the mass of potential entrants, M_{ei}^c, to be such that at any instant the mass of successful entrants, $[1 - G(t_i^{*c})] M_{ei}^c$, equals the mass of incumbent firms who die, δM_i^c:

$$[1 - G(t_i^{*c})] M_{ei}^c = \delta M_i^c \tag{58}$$

Second, the free entry condition equates the entry cost, F_{ei}, to the expected profit prior to entry, $\overline{\pi}_i^c$, discounted by the probability of death and multiplied by the probability of successful entry:

$$[1 - G(t_i^{*c})] \overline{\pi}_i^c / p_{death} = F_{ei} \tag{59}$$

Third, equilibrium in the goods markets requires

$$\begin{aligned} M_{YA}\overline{s}_Y^A + M_{YB}\overline{s}_Y^B &= \varepsilon_Y (E_A + E_B) & (60) \\ M_{ZA}\overline{s}_Z^A + M_{ZB}\overline{s}_Z^B &= \varepsilon_Z (E_A + E_B) & (61) \\ M_{YA}\overline{s}_Y^A + M_{ZA}\overline{s}_Z^A &= E_A & (62) \\ M_{YB}\overline{s}_Y^B + M_{ZB}\overline{s}_Z^B &= E_B & (63) \end{aligned}$$

Forth, equilibrium in factor markets requires that factor demand plus unemployment equal to factor supply

$$\begin{aligned} \overline{l}_Y^c (t_Y^{*c}) M_Y^c + \overline{l}_Z^c (t_Z^{*c}) M_Z^c &= \nu_L^c \overline{L}; \qquad c = A, B. & (64) \\ \overline{h}_Y^c (t_Y^{*c}) M_Y^c + \overline{h}_Z^c (t_Z^{*c}) M_Z^c &= \nu_k^c \overline{K}; \qquad c = A, B. & (65) \end{aligned}$$

In writing the factor market equilibrium equations we have taken into account that the stationarity condition implies that the quantity of each factor released by firms who die is equal to the quantity of each factor demanded by successful entrants. After replacing average profit, average sales, average factor demands, and cut off output in (59)-(65) the general equilibrium system counts eleven independent equations and twelve endogenous variables. The equations are the four free entry conditions (59), three out of four goods market equilibrium conditions (60)-(63), the four factor market equilibrium conditions (64)-(65). The endogenous are the four masses $\{M_Z^c\}$, the four outside options $\{w_{Lo}^c, w_{Ho}^c\}$ and the four cut off values $\{t_i^{*c}\}$. The choice of a numéraire makes the system determined.

12.4 Ranking of cut off values

Using (53) and (54) we may write the free entry condition (59) as

$$\int_{t_i^{*c}}^{\infty} \left[\left(\frac{q_i^c(t)}{q_i^c(t_i^{*c})} \right)^{\frac{\varsigma - 1}{\varsigma}} - 1 \right] dG = \frac{F_{ie} p_{death}}{F_i}. \tag{66}$$

After replacing the optimal sampling, screening, and factor inputs into (66) we obtain

$$\underbrace{\int_{t_i^{*c}}^{\infty}\left[\left(\frac{(\Phi_i)^{\frac{-\delta}{\Delta}}(\omega_o^c)^{\omega_5}(\beta)^{\flat_0}+(\alpha)^{\flat_0}}{(\Phi_i)^{\frac{-\delta}{\Delta}}(\omega_o^c)^{\omega_5}(\beta_i^{*c})^{\flat_0}+(\alpha_i^{*c})^{\flat_0}}\right)^{\frac{(\varsigma-1)\sigma}{\varsigma(\sigma-1)}}-1\right]dG}_{\Upsilon\left(\Phi_i,\omega_o^c,t_i^{*c}\right)}=\frac{F_{ei}p_{death}}{F_i} \tag{67}$$

where $\flat_0=\frac{-\mathfrak{s}\delta\gamma}{\Delta}>0$, $\omega_5=\frac{(\gamma(\delta-\chi)+1)\mathfrak{s}}{\Delta}<0$. By inspection of the left hand side of equation (66) it is immediate that

$$\Upsilon'_{\Phi_i}\left(\Phi_i,\omega_o^c,t_i^{*c}\right) \gtreqless 0 \Leftrightarrow \flat(t) \gtreqless \flat\left(t_i^{*c}\right) \tag{68}$$

$$\Upsilon'_{\omega_o^c}\left(\Phi_i,\omega_o^c,t_i^{*c}\right) \lesseqgtr 0 \Leftrightarrow \flat(t) \gtreqless \flat\left(t_i^{*c}\right) \tag{69}$$

$$\Upsilon'_{t_i^{*c}}\left(\Phi_i,\omega_o^c,t_i^{*c}\right) < 0 \tag{70}$$

Therefore

$$\frac{dt_i^{*c}}{d\Phi_i} = -\frac{\Upsilon'_{\Phi_i}\left(\Phi_i,\omega_o^c,t_i^{*c}\right)}{\Upsilon'_{t_i^{*c}}\left(\Phi_i,\omega_o^c,t_i^{*c}\right)} \gtreqless 0 \Leftrightarrow \flat(t) \gtreqless \flat\left(t_i^{*c}\right) \tag{71}$$

$$\frac{dt_i^{*c}}{d\omega_o^c} = -\frac{\Upsilon'_{\omega_o^c}\left(\Phi_i,\omega_o^c,t_i^{*c}\right)}{\Upsilon'_{t_i^{*c}}\left(\Phi_i,\omega_o^c,t_i^{*c}\right)} \lesseqgtr 0 \Leftrightarrow \flat(t) \gtreqless \flat\left(t_i^{*c}\right) \tag{72}$$

Recall that heterogeneity is *H-biased* if $\flat'(t)>0$, neutral if $\flat'(t)=0$, and *L-biased* if $\flat'(t)<0$ for any t. Equivalently, we may say that heterogeneity is *H-biased* if $\flat(t)>\flat\left(t_i^{*c}\right)$, neutral if $\flat(t)=\flat\left(t_i^{*c}\right)$, and *L-biased* if $\flat(t)<\flat\left(t_i^{*c}\right)$ for any $t>t_i^{*c}\geqslant t_0$. Thus, equation (71) allows ranking $\{t_Y^{*c},t_Z^{*c}\}$ and equation (72) $\left\{t_i^{*A},t_i^{*B}\right\}$ according to the heterogeneity bias. This result is used in the next section.

12.5 No factor intensity reversal.

Let us rewrite the two factor intensity more succinctly:

$$\theta_i^c = (\Phi_i)^{\frac{\chi-\delta}{\Delta}}\left(\frac{w_{Ho}^c}{w_{Lo}^c}\right)^{\frac{(\delta-\chi)}{\Delta}}[\flat(t)]^{-\frac{\gamma\mathfrak{s}(\delta-\chi)}{\Delta}} \tag{73}$$

$$\overline{\theta}_i^c = (\Phi_i)^{\frac{\chi-\delta}{\Delta}}\left(\frac{w_{Ho}^c}{w_{Lo}^c}\right)^{\frac{(\delta-\chi)}{\Delta}}\left[\widetilde{\flat}_{\theta,i}^c\right]^{-\frac{\gamma\mathfrak{s}(\delta-\chi)}{\Delta}} \tag{74}$$

where

$$\widetilde{\flat}_{\theta,i}^c=\left[\frac{1}{1-G\left(t_i^{*c}\right)}\int_{t_i^{*c}}^{\infty}[\flat(t)]^{-\frac{\gamma\mathfrak{s}(\delta-\chi)}{\Delta}}g(t)\,dG\right]^{-\frac{\Delta}{\gamma\mathfrak{s}(\delta-\chi)}} \tag{75}$$

from which

$$\frac{d\widetilde{\flat}_{\theta,i}^{c}}{dt_i^{*c}} \gtreqless 0 \Leftrightarrow \flat'(t) \gtreqless 0. \tag{76}$$

Then,

$$\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \underbrace{\left(\frac{\Phi_Y}{\Phi_Z}\right)^{\frac{\chi-\delta}{\Delta}}}_{>1} \underbrace{\left(\frac{\widetilde{\flat}_{\theta,Y}^{c}}{\widetilde{\flat}_{\theta,Z}^{c}}\right)^{-\frac{\gamma\mathfrak{s}(\delta-\chi)}{\Delta}}}_{>1} > 1 \tag{77}$$

which shows that selection into entry just reinforce the effect of Φ_i on average factor intensity.