

<b>Instructor's name</b>	Antonio Attalenti
<b>BA</b>	Business Economics
<b>Academic year</b>	2015/16
<b>Term</b>	First
<b>Credits</b>	10
<b>Subject area</b>	SECS-S06

**Course of Mathematics for Business and Economics (a.a. 2015/2016)**

**(Prof. Antonio Attalenti )**

University of Bari Aldo Moro

BA Business Economics

Admission criteria

Algebraic elementary calculus and basics of analytic geometry (equation of a straight line and related topics)

Aims of the course

Providing the main mathematical tools frequently occurring in problems related to business administration, economics and finance; developing suitable techniques of quantitative analysis to face up problems of evaluation and choice in the same areas.

Course outline

Basics on set theory: logical symbols, sets, elements and related properties. Set operations: union, intersection, difference, complement, symmetric difference. Cartesian product. The numerical sets  $\mathbf{N}$ ,  $\mathbf{Z}$ ,  $\mathbf{Q}$  and related properties.

The set  $\mathbf{R}$  of real numbers: algebraic and order properties. Upper and lower bound of a subset of  $\mathbf{R}$ . Bounded and unbounded sets. Maximum and minimum, supremum and infimum of a subset of  $\mathbf{R}$ . The completeness property and equivalent versions. Some applications:  $n$ -roots, exponentials and logarithms. Absolute value, integer part and fractional part of a real number. Intervals of  $\mathbf{R}$ . The density of  $\mathbf{Q}$  in  $\mathbf{R}$ . The extended real line: neighborhoods, cluster points and isolated points.

Functions: domain, range and graph. Injective, surjective, bijective and invertible functions. Composition of two or more functions. Inverse function. Restrictions of a function. Real functions of one real variable: upper and lower bound, supremum and infimum, maximum and minimum. Local and global extrema. Bounded, odd, even, periodic, monotone and convex functions. Sequences of real numbers. Sequences defined by recurrence. Arithmetic and geometric progressions with applications: simple and continuous compounding in finance. The factorial of a natural number. The study of some elementary functions: constant function, identity function, affine function, piecewise affine function, absolute value function, power function,  $n$ -root function, exponential function, logarithmic function, power function with real exponent, trigonometric functions and the corresponding inverse functions. Equations and inequations. Determining the domain of a function.

Limits: basic definitions and corresponding interpretation. Limit of sequences. **Uniqueness of the limit**. Local character of the limit. Limit of a restriction of a function. Non – regularity test. Right-hand and left-hand limit and related theorem. Comparison theorems. **Squeeze theorem**. Divergence criterion. Operations with the limits. Indeterminate forms. Limit of the composition of functions. Theorem about the limit of monotone functions/sequences. Limits of the elementary functions. Some fundamental limits. Neper's number and its financial meaning. Asymptotic analysis for computing limits in indeterminate forms and Landau's symbols. An estimate of the growth of  $n!$ : De Moivre-Stirling's formula.

Continuity: definition of the continuity of a function at a point and basic properties. Points of discontinuity and the corresponding classification. Integer part and fractional part functions and the related discontinuities. Functions everywhere continuous in their domains. Sum, product, quotient and composition of continuous functions. Continuity criterion for monotone functions. Continuity of the elementary functions. Intermediate value property and Bolzano's theorem. **Existence of zeros theorem, fixed point theorem and Weierstrass's theorem**.

Differentiation: the concept of derivative and its meaning in different frameworks. Differentiable functions Left and right derivative. Geometric interpretation: tangent line and rate of approximation. Angular and cusp points. **Continuity of the differentiable functions**. Differentiation rules. Higher order derivatives and Lagrange's spaces. The chain rule and the differentiability of the inverse function Determining the derivatives of the elementary functions. Elasticity, semielasticity and applications in Economics and Finance.

Applications of the differential calculus: functions which are strictly monotone at a point: necessary condition and sufficient condition. Local extrema. Stationary points. **Fermat's theorem. Main theorems in differentiation: Rolle's theorem** Cauchy's theorem and Lagrange's theorem. Darboux's theorem. **Consequences of Lagrange's theorem**. Monotonicity test for differentiable functions. Some sufficient conditions for local extrema. Convexity/concavity test through the sign of the second derivative. Inflection points: a necessary condition and some sufficient conditions. De L'Hospital's rule and applications for computing limits in indeterminate forms. Discontinuity of the first derivative. Second order Taylor's expansion and some applications. Asymptotes and graph-sketching.

Basics of integration theory: antiderivatives, **indefinite integral and main properties**. The standard rules of integration. Integration by parts and by substitution. Riemann lower and upper integral sums. The Riemann integrability and the corresponding integral. Criterion of integrability and the integral as a limit. Properties of the definite integrals. Computing areas of normal domains. Some classes of integrable functions: the integrability of continuous functions and of monotone functions. **Mean value theorem**. Torricelli-Barrow's theorem. **Newton-Leibnitz's theorem (or Fundamental theorem of integral calculus)**.

Some elements of linear algebra: vectors in  $\mathbf{R}^n$  and basic operations. Linearly independent vectors and basis in Euclidean spaces. Matrices, determinants and related properties. The rank of a matrix. Kronecker's theorem. Solving systems of linear equations: Cramer's formula and Rouchè-Capelli's theorem.

Functions of two variables: graph, coordinate lines and level curves. Cobb-Douglas functions in Economics. Limits and continuity. Partial derivatives and gradient vector. Differentiability and tangent plane. The chain rule. Directional derivatives and the gradient formula. Some properties of the vector gradient. Second-order partial derivatives and Schwarz's theorem. Hessian matrix.

Unconstrained optimization. Something about constrained optimization: Lagrange multipliers and their economic meaning.

The students have to know all the definitions and statements of the theorems indicated above; the proof is required in addition if the theorem is marked in boldface.

Reading material

Slides available on line at [www.uniba.it/ricerca/dipartimenti/disag](http://www.uniba.it/ricerca/dipartimenti/disag)

Other references

- 1) M. Bramanti, C. D. Pagani, S. Salsa, *Matematica: calcolo infinitesimale e algebra lineare*, Zanichelli, Bologna.
  - 2) S. Salsa, A. Squellati, *Esercizi di Matematica: calcolo infinitesimale e algebra lineare, Volume I*, Zanichelli, Bologna.
  - 3) P. Marcellini, C. Sbordone, *Esercitazioni di Matematica, Volume I, Parte prima e seconda*, Liguori Editore, Napoli.
  - 4) A. Attalienti, S. Ragni, *Esercitazioni di Matematica*, Giappichelli, Torino.
- 

Assessment methods

- Assignment: No
- Written without oral presentation: No
- Oral presentation: Yes
- 

Tutorials

- This course is in e-learning Web Site area: No

Teaching methods

- Direct contact
  - Lectures: Yes
  - Tutorials: Yes
  - Personal work
  - Case studies – in group: No