PROGRAM OF "CALCULUS 2"

Numerical series

Convergent, divergent, indeterminate series. Necessary conditions for the convergence of a series. Sums and products of series. Geometric series. Series with non-negative terms. Convergence criteria: comparison test, ratio test, root test. Harmonic series. Absolute convergence of series. Cauchy criterion. Alternating series: Leibnitz criterion.

Series of function

Convergence of series of functions. Power series. Set and radius of convergence: properties. D'Alambert and Cauchy-Hadamard criteria for the calculus of the radius of convergence. Properties of the sum of a power series. Taylor series. Sufficient condition for the expansion of a function in Taylor series.

Elements of linear algebra

Vector spaces and subspaces. Linear combination of vectors. Linear independence. Spanning sets. Bases of finite dimensional vector spaces. Linear maps. Matrices: sum and scalar product of matrices. Vector space of matrices. Multiplication of matrices. Inverse of a square matrix. Determinant of square matrices: properties. Laplace's formula. Rank of a matrix. Systems of linear equations. Cramer's rule. Rouché-Capelli theorem. Methods for solving linear systems.

Functions of several variable and differentiation

Topological properties of the Euclidean space**.** Metric on Euclidean space and related properties. Norm and scalar product. Neighbourhoods, open and closed sets. Interior, limit, boundary and isolated points of a set. Bounded sets, compact sets and connected sets. Limits of functions. Continuous functions: properties and main theorems. Partial and directional derivatives. Differentiable functions: properties. Continuously differentiable functions, sufficient condition for the differentiability of a function. Gradient vector. Higher order partial derivatives. Schwarz theorem. Chain rule for differentiable functions. Lagrange theorem. Functions with zero gradient. Hessian matrix. Local extrema for functions of several variables. Absolute minima and maxima.

Riemann integration for several variable functions

Peano-Jordan measure of sets and related properties. Riemann integrable functions: properties. Interchange of the order of integration in double integrals. Volumes. Double integrals in polar coordinates. Change of coordinates.

Curves and differential forms

Parametric equation of a line. Curves in $\boldsymbol{R}^n$: properties. Rectifiable curves, length of curves. Line integral of scalar functions: properties. Differential forms and vector field. Line integral of vector fields. Conservative fields. Potential in conservative fields. Independence of path. Fundamental theorem of calculus for line integrals of vector fields. Gauss-Green theorem.

Ordinary differential equations

Basic concepts. Cauchy problem. Existence and uniqueness theorem for ordinary differential equations in normal form. Systems of first order differential equations. Reduction of *n*-th order differential equations into first order systems of *n* equations. Linear systems of ordinary differential equations: existence and uniqueness theorem, vector space spanned by the set of all solutions. Wronskian. Dimension of the vector space spanned by all solutions of a homogenous system of linear ordinary differential equations and fundamental set of solutions. Set of solutions of non-homogenous system of first order linear differential equation and Lagrange undetermined coefficients method. Linear differential *n*-order equations with constant coefficients: characteristic polynomial. Methods for solving the following first order differential equations: separable, homogenous, linear, Bernoulli and exact equations. Methods for solving second order differential equations which may be reduced to first order differential equations. Resolutions methods for linear differential *n*-order equations with constant coefficients.