## Prova scritta di Matematica per l'Economia L-Z-11 Aprile 2022

1. Studiare e tracciare il grafico della funzione:

$$f(x) = \ln\left|\frac{x-3}{x+2}\right|$$

2. Approssimare con il polinomio di Taylor di grado $n = 2$ e punto iniziale $x_0 = 0$ la funzione:

$$f(x) = e^{x+\sin x}$$

3. Calcolare il seguente integrale:

$$\int x^{-\frac{3}{2}} \cdot \ln(1+x)\, dx$$

4. Studiare il sistema $Ax = b$ al variare del parametro $k \in \mathbb{R}$:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & k & 1 \\ k & -k & 4 \end{pmatrix}; \quad b = \begin{pmatrix} 1 \\ 0 \\ k \end{pmatrix}; \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix};$$

Soluzione

1) $f(x) = \ln\left|\frac{x-3}{x+2}\right|$ ; D : $\begin{cases} \left|\frac{x-3}{x+2}\right| > 0 \quad \nless = 0 \\ x+2 \neq 0 \end{cases}$

$$\begin{cases} x-3\neq 0 & x\neq 3 \\ x\neq -2 \end{cases} \qquad \mathbb{R}-\{-2;3\}$$

Pos. $\ln\left|\dfrac{x-3}{x+2}\right|>0 \Leftrightarrow \left|\dfrac{x-3}{x+2}\right|>1$

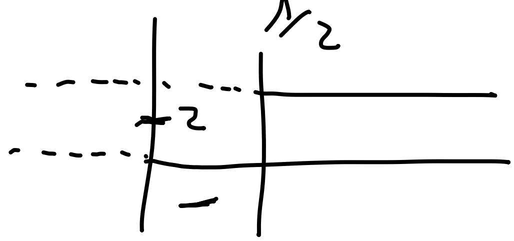
$$\frac{x-3}{x+2}>1 \quad \cup \quad \frac{x-3}{x+2}<-1$$

$$\frac{x-3-x-2}{x+2}>0 \quad \cup \quad \frac{x-3+x+2}{x+2}<0$$

$$\frac{-5}{x+2}>0 \quad \cup \quad \frac{2x-1}{x+2}<0$$

$$\Downarrow$$

$$x<-2 \quad \cup$$




$$-2<x<\frac{1}{2}$$

$f(x)>0 \quad ]-\infty,-2[\ \cup\ ]-2,\frac{1}{2}[$

$f(x)=0 \quad x=\frac{1}{2}$

$f(x)<0 \quad ]\frac{1}{2},+\infty[-\{3\}$

$$\lim_{x \to \pm\infty} \ln\left|\frac{x-3}{x+2}\right| = \ln 1 = 0$$

$$\lim_{x \to -2} \ln\left|\frac{-5}{0}\right| = \ln\left(\frac{5}{0^+}\right) = +\infty$$

$$\lim_{x \to +3} \ln\left|\frac{x-3}{x+2}\right| = \ln 0 = -\infty$$

$$f'(x) = \frac{1}{\cancel{\left|\frac{x-3}{x+2}\right|}} \cdot \frac{\cancel{\left|\frac{x-3}{x+2}\right|}}{\left(\frac{x-3}{\cancel{x+2}}\right)} \cdot \frac{x+2-x+3}{(x+2)^{\cancel{2}}} =$$

$$\frac{5}{(x+2)(x-3)} = \frac{5}{x^2-x-6}$$

$f'(x) > 0$ ——— $-2$ - - - - - $3$ ———

↗ ↘ ↗

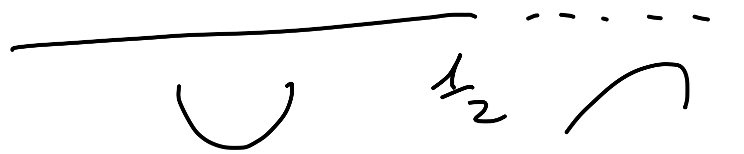
$f'(x) = 0$ MAI

No max e min relativi

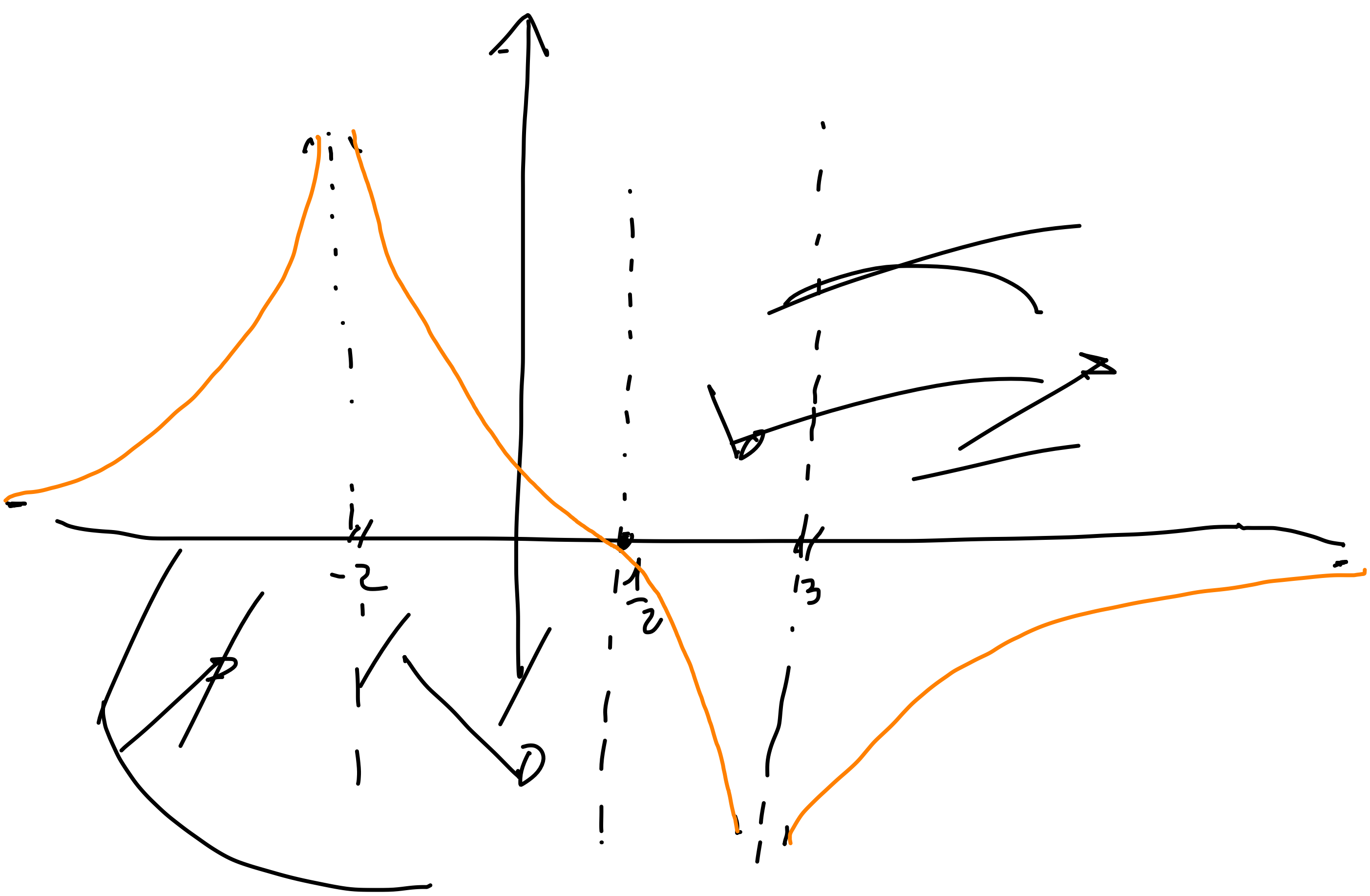
$$f''(x) = \frac{-10x+5}{(x^2-x-6)^2}$$

$$f''(x) > 0 \qquad -10x+5>0$$

$$\cancel{10}\,x < \frac{5}{10} \Rightarrow x < \frac{1}{2}$$




$$f''(x) = 0 \qquad x = \frac{1}{2}$$



2) $e^{x+\sin x}$ ; $f'(x) = e^{x+\sin x} \cdot (1+\cos x)$

$f(0) = 1$ ; $f'(0) = 2$

$f''(x) = e^{x+\sin x}(1+\cos x)^2 + e^{x+\sin x} \cdot -\sin x$

$= e^{x+\sin x}\left[(1+\cos x)^2 - \sin x\right]$

$f''(0) = 1 \cdot [4 - 0] = 4$

$f(x) \approx 1 + 2x + 4 \cdot \frac{(x)^2}{2}$

$\approx 1 + 2x + 2x^2$

3) $\int \underbrace{x^{-\frac{3}{2}}}_{f'} \cdot \underbrace{\ln(1+x)}_{g} \, dx$ PARTI

$f = \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}$

$= -\frac{2}{\sqrt{x}}$ ; $g' = \frac{1}{1+x}$

$\int x^{-\frac{3}{2}} \ln(1+x) \, dx = -\frac{2}{\sqrt{x}} \cdot \ln(1+x) - \int -\frac{2}{\sqrt{x}} \cdot \frac{1}{1+x}$

$$= -\frac{2}{\sqrt{x}}\ln(1+x) + \int \frac{2}{\sqrt{x}\cdot(1+x)}\,dx$$

$$\sqrt{x} = t$$

$$x = t^2; \quad dx = 2t\,dt$$

$$= \int \frac{2}{\cancel{t}\cdot(1+t^2)}\,2\cancel{t}\,dt = 4\int \frac{1}{1+t^2}\,dt$$

$$= 4\,\mathrm{arctg}\,t = 4\,\mathrm{arctg}\sqrt{x}$$

4)

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & k & 1 \\ k & -k & 1 \end{pmatrix};$$

$$|A| = 1\cdot\begin{vmatrix} k & 1 \\ -k & 1 \end{vmatrix} + 1\cdot\begin{vmatrix} -1 & k \\ k & -k \end{vmatrix} = (k+k)+(k-k^2)$$

$$= 3k - k^2$$

$$k(3-k)$$

$$|A| \neq 0 \quad k \neq 0;\ k \neq 3$$

$$\mathrm{car}(A) = 3 \text{ se } k \neq 0;\ k \neq 3$$

$x_1 = \frac{k-2}{k-3}$ ; $x_2 = \frac{k-1}{k(k-3)}$ ; $x_3 = \frac{1}{3-k}$

Se $k = 0$ :

$B = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ ; $B^* = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$|B^*| = \cancel{+}1 \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} = -1 \neq 0$ Car$(B) = 3$

(SIST. IMPOS)

$k = 3$

$B = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 3 & 1 & 0 \\ 3 & -3 & 1 & 3 \end{pmatrix}$ ; $B^* = \begin{pmatrix} 0 & 1 & 1 \\ 3 & 1 & 0 \\ -3 & 1 & 3 \end{pmatrix}$

$|B^*| = -1 \cdot \begin{vmatrix} 3 & 0 \\ -3 & 3 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} = -9 + (3+3)$
$= -3 \neq 0$

Car$(B) = 3$ SIST IMPOSS