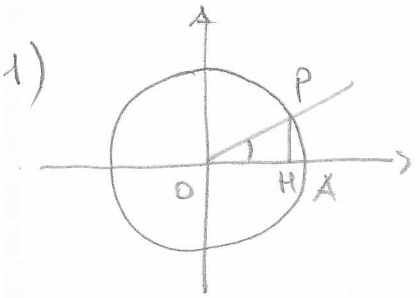
SIN E COS di angoli notevoli

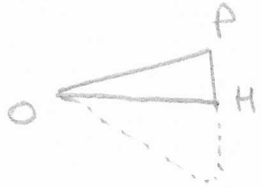
1)



$\alpha = \frac{\pi}{6}$ $(30°)$

$\overline{PH} = \sin\alpha$, $\overline{OH} = \cos\alpha$

$O\hat{P}H = 60°$



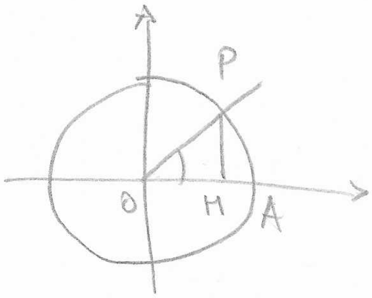
$O\overset{\triangle}{P}H$ è la metà di un triangolo equilatero di lato $\overline{OP} = 1$

$$\overline{OH} = \overline{OP} \cdot \frac{\sqrt{3}}{2} = 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\overline{PH} = \frac{\overline{OP}}{2} = \frac{1}{2}$$

Quindi $\sin(\pi/6) = \frac{1}{2}$, $\cos(\pi/6) = \frac{\sqrt{3}}{2}$

2



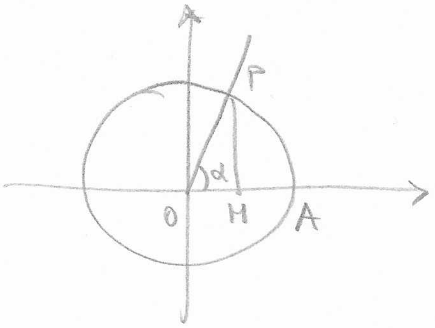
$\alpha = \frac{\pi}{4}$ $(45°)$

$P\hat{O}H = O\hat{P}H = 45°$

$O\overset{\triangle}{P}H$ è la metà di un quadrato la cui diagonale $\overline{OP}$ misura 1

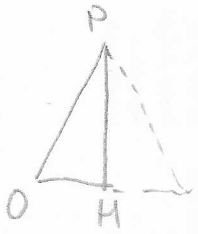
$$\overline{OH} = \cos(\alpha) = \overline{PH} = \sin\alpha = \frac{\overline{OP}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

3



$\alpha = \frac{\pi}{3}$ $(= 60°)$

$\overline{PH} = \sin\alpha$, $\overline{OH} = \cos\alpha$

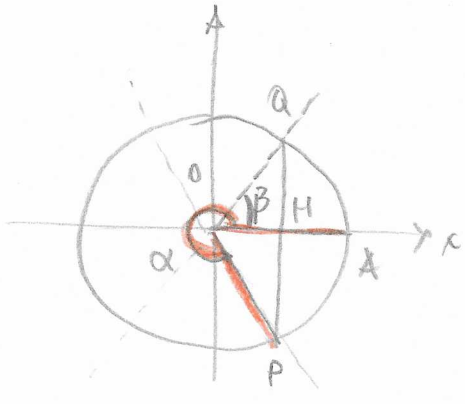


$O\overset{\triangle}{P}H$ è metà di un tr. equil. di lato $\overline{OP}$ pari ad 1

$$\overline{PH} = \frac{\overline{OP}}{2} \cdot \sqrt{3} = \frac{\sqrt{3}}{2}, \quad \overline{OH} = \frac{\overline{OP}}{2} = \frac{1}{2} \implies \sin\alpha = \frac{\sqrt{3}}{2}, \ \cos\alpha = \frac{1}{2}$$

Per altri angoli ci si riconduce ai 3 casi precedenti

ad es: $\alpha = \frac{5\pi}{3} = \frac{3}{3}\pi + \frac{2}{3}\pi$



ci si riconduce a $\beta = \pi/3$

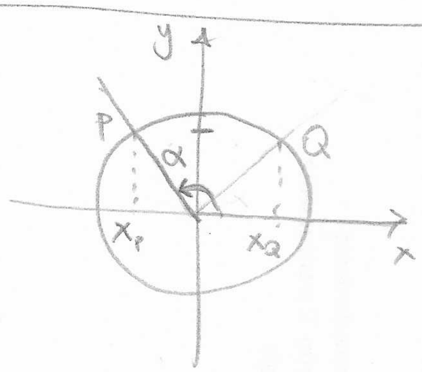
Infatti, $\overline{QH} = \overline{HP}$, e i triangoli OQH e OHP sono congruenti e simmetrici rispetto Ox.

$\sin\alpha = -\sin\beta$ (i segmenti QH ed HP hanno stessa lungh. ma le ordinate di Q e P hanno segno opposto)

⇓

$\sin\frac{5}{3}\pi = -\frac{\sqrt{3}}{2}$; $\cos\alpha = \cos\beta = \frac{1}{2}$ (Q e P hanno la stessa ascissa)

$\alpha = \frac{3}{4}\pi$



Ci si riconduce all'angolo $\beta = \frac{\pi}{4}$. Infatti: $x_P = -x_Q$ mentre $y_P = y_Q$ (P e Q hanno le stesse ordinate)

Quindi $\sin\alpha = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\alpha = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$