

$$\lim_{x \rightarrow 0} \frac{\arcsin x - x}{\lg x^2(1 - \cos 2x) - \sin(\log(1 + 3x^3))} =$$

$$= \lim_{x \rightarrow 0} \frac{\arcsin x - x}{\frac{\lg x^2(1 - \cos 2x)}{x^2 \cdot 4x^2} \cdot 4x^4 - \frac{\sin(\log(1 + 3x^3))}{\log(1 + 3x^3)} \cdot \frac{\log(1 + 3x^3)}{3x^3} \cdot 3x^3} =$$

$$= \lim_{x \rightarrow 0} \frac{\arcsin x - x}{\left[ \frac{\lg x^2}{x^2} \cdot \frac{1 - \cos 2x}{4x^2} \cdot 4x^4 - \frac{\sin(\log(1 + 3x^3))}{\log(1 + 3x^3)} \cdot \frac{\log(1 + 3x^3)}{3x^3} \cdot 3 \right] x^3} \quad (*)$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $1 \cdot \frac{1}{2} \cdot 4 \quad 1 \quad 1 \quad 3$

solo a parte il  $\lim_{x \rightarrow 0} \frac{\arcsin x - x}{x^3} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}} - 1}{3x^2} =$

$$= \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{3x^2 \sqrt{1-x^2}} = \left\{ \begin{array}{l} \text{divido e moltiplico} \\ \text{per } 1 + \sqrt{1-x^2} \end{array} \right\} =$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \sqrt{1-x^2})(1 + \sqrt{1-x^2})}{3x^2 \sqrt{1-x^2} (1 + \sqrt{1-x^2})} = \lim_{x \rightarrow 0} \frac{1 - (1-x^2)}{3x^2 \sqrt{1-x^2} (1 + \sqrt{1-x^2})} =$$

$\downarrow \quad \downarrow$   
 $1 \quad 2$

$$= \lim_{x \rightarrow 0} \frac{x^2}{3x^2 \cdot 2} = \frac{1}{6} \quad \text{— Dunque, passando al limite in (*) si ha:}$$

$$\frac{\frac{1}{6}}{0 - 3} = - \frac{1}{18}$$

$$\begin{aligned} \int x^2 \log x \, dx &= \frac{x^3}{3} \log x - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \frac{x^3}{3} \log x - \frac{1}{3} \int x^2 \, dx = \\ &= x^3 \log x - \frac{x^3}{9} + c \end{aligned}$$

$$\begin{aligned} \int x^2 \cos x \, dx &= x^2 \sin x - \int 2x \sin x \, dx = x^2 \sin x - 2 \left( -x \cos x + \right. \\ &\left. + \int \cos x \, dx \right) = x^2 \sin x + 2x \cos x - 2 \sin x + c \end{aligned}$$

PER ITERAZIONE

$$\begin{aligned} \int \sin(\log x) \, dx &= x \sin(\log x) - \int x \frac{\cos(\log x)}{x} \, dx = x \sin(\log x) \\ &- \left( x \cos(\log x) + \int x \sin(\log x) \, dx \right) = \\ &= x \left[ \sin(\log x) - \cos(\log x) \right] - \int x \sin(\log x) \, dx \end{aligned}$$

$$\Leftrightarrow 2 \int \sin(\log x) \, dx = x \left[ \sin(\log x) - \cos(\log x) \right] + c$$

$$\Leftrightarrow \int \sin(\log x) \, dx = \frac{x}{2} \left[ \sin(\log x) - \cos(\log x) \right] + c$$

$$\begin{aligned} \int x^2 e^{-x} \, dx &= -x^2 e^{-x} + \int x e^{-x} \, dx = -x^2 e^{-x} + 2 \left( -x e^{-x} + \int e^{-x} \, dx \right) = \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c = \\ &= -e^{-x} (x^2 + 2x + 2) + c \end{aligned}$$