

calcolarne la derivata e dire per quali valori di x essa non esiste.

$$\left[x \neq \frac{\pi}{2} + 2k\pi; y' = \frac{\cos x}{|\cos x|} \cdot \frac{1}{1 - \sin x}; x \neq \frac{\pi}{2} + k\pi \right]$$

13. Calcolare i punti di intersezione delle due curve

Tangenti a curve di data equazione

Scrivere l'equazione della tangente al grafico di ciascuna delle seguenti curve nei punti la cui ascissa è indicata a fianco fra parentesi:

1. $y = 4x^3$. (x = -1) 2. $y = 5x^2 + 3x$. (x = 1)

3. $y = 3x^3$. (x = 2) 4. $y = 2x^2 + 2x - 3$. (x = 1)

5. $y = 4x^2 - 5x + 1$. (x = -1) 6. $y = x^2 + \frac{1}{x}$. (x = 2)

7. $y = \frac{x+2}{x}$. (x = -1) 8. $y = \sqrt{x-2}$. (x = 6)

9. $y = \frac{2x-1}{3x+2}$. (x = $\frac{1}{2}$) 10. $y = \frac{2x-5}{3x-1}$. (x = -3)

11. $y = \sqrt{3x^2 - 2}$. (x = -3) 12. $y = \sin x + \cos x$. (x = $\frac{\pi}{2}$)

13. $y = \sin x - \cos x$. (x = 0) 14. $y = x + \sin x$. (x = 0)

15. $y = x - \cos x$. (x = 0) 16. $y = e^x$. (x = 0)

17. $y = 3 \cos x - \tan x$. (x = 0)

18. $y = 2 \cos^2 x - 3x^2 + 9x - \sin x$. (x = 0)

19. $y = \frac{\sin^2 x}{3 \sin x - \cos x}$. (x = $\frac{\pi}{4}$)

20. $y = \frac{e^x \ln x}{x}$. (x = 1)

21. $y = \arcsin\left(-\frac{x}{2}\right) + \arctan x$. (x = $\sqrt{3}$)

22. $y = \frac{2x + \sin x + \cos x}{x + \cos x}$. (x = 0)

23. $y = e^{\sqrt{3}\sin x + \cos x + 1}$. (x = 0)

esercizi 5

26. $y = \operatorname{arc tg} \frac{\sin x}{\sqrt{1+\cos^2 x}} + \ln(\cos x + \sqrt{1+\cos^2 x})$. $\left[\frac{\cos x - \sin x}{\sqrt{1+\cos^2 x}} \right]$
27. $y = \log \left(1 + \operatorname{tg} \frac{x}{2} \right)$. $\left[\frac{1}{\sin x + \cos x + 1} \right]$
28. $y = x \operatorname{arc tg} x - \log \sqrt{1+x^2}$. $[\operatorname{arc tg} x]$
29. $y = \operatorname{arc tg} \frac{\sin x}{1+\cos x}$. $\left[\frac{1}{2} \right]$
30. $y = \log \frac{x-1}{\sqrt{x^2+x+1}} - \sqrt{3} \operatorname{arc tg} \frac{2x+1}{\sqrt{3}}$. $\left[\frac{3}{x^3-1} \right]$
31. $y = \log \frac{\sqrt{1+x^2}}{x} - \frac{1}{2x^2}$. $\left[\frac{1}{x^3(x^2+1)} \right]$
32. $y = e^x \log \cos \sqrt{x}$. $\left[e^x \left\{ \log \cos \sqrt{x} - \frac{1}{2\sqrt{x}} \operatorname{tg} \sqrt{x} \right\} \right]$
33. $y = x \operatorname{arc sen} \sqrt{x} - \frac{1}{2} \operatorname{arc tg} \sqrt{\frac{x}{1-x}} + \frac{1}{2} \sqrt{x(1-x)}$. $[\operatorname{arc sen} \sqrt{x}]$
34. $y = \sqrt{1-e^{2\cos x}} + \operatorname{arc sen} \sqrt{1-e^{2\cos x}}$. $\left[\frac{\sin x \cdot e^{\cos x} (e^{\cos x} + 1)}{\sqrt{1-e^{2\cos x}}} \right]$
35. $y = \operatorname{arc tg} \left(1 - \frac{4x}{x^2+2x-1} \right)$. $\left[\frac{2}{1+x^2} \right]$
36. $y = \operatorname{arc tg} \frac{3 \operatorname{tg} 2x + 4}{3 - 4 \operatorname{tg} 2x}$. non è derivabile la funzione $y = [\operatorname{sen} x]?$ Perché? [2]
37. $y = \operatorname{arc tg} \frac{3x-x^3}{1-3x^2}$. $\left[\frac{3}{x^2+1} \right]$
38. $y = \operatorname{arc tg} \frac{\operatorname{sen} x}{\cos^2 x}$. $\left[\frac{\cos x(2-\cos^2 x)}{\cos^4 x - \cos^2 x + 1} \right]$
39. $y = \operatorname{arc tg} \frac{2}{\cos^2 2x}$. privata della funzione $y = \sqrt{x^2+3x^2}$ dopo aver calcolato la derivata destra in $x=0$ e le derivate sinistra in $x=-3$ e destra in $x=3$, non è derivabile.
40. $y = \operatorname{arc tg} \frac{1}{\sqrt{x^2-1}} + \sqrt{x^2-1}$. $\left[\frac{\sqrt{x^2-1}}{x} \right]$
41. $y = \operatorname{arc tg}(1+2\cos x) - \operatorname{arc tg} \frac{\cos x}{1+\cos x}$. non è derivabile la funzione $y = [\operatorname{tg} x]?$ Perché? [0]

$$\lim_{x \rightarrow 0} \frac{3x^2 - \log_4(1 - \sin^3 4x)}{4 \arcsin^2(x + 2 \operatorname{tg} 5x)} = \frac{3}{184}$$

$$\lim_{x \rightarrow 0} \frac{3x^3 + 1 - \cos 6x \sqrt{x} - x^6}{2 \arctg(3^x - 1) \log(1 + \sin 6x)} = \frac{11}{12 \log 3}$$

$$\lim_{x \rightarrow +\infty} \frac{5^{\frac{2}{x^2}} - 1 + \operatorname{tg} \frac{1}{3x^2}}{2 \arctg \frac{9}{x^3} - \sin \frac{1}{x^2}} \quad (\text{pone } y := \frac{1}{x}) = -(2 \log 5 + \frac{1}{3})$$

$$\lim_{x \rightarrow 0} \frac{4 - 4 \cos \frac{x}{2} - 3 \log \frac{x^2}{2}}{\arctg(2x \sin 5x) + 2 \operatorname{tg}^3 x} = -\frac{1}{10}$$

$$\lim_{x \rightarrow +\infty} \frac{\arcsin^3 \frac{1}{\sqrt{x}} - \operatorname{tg}^2 \left(\log_2 \left(1 + \frac{1}{x^3} \right) \right)}{\sqrt{x} \sin \frac{9}{x}} = \frac{1}{9}$$

$(y := \frac{1}{\sqrt{x}} \Rightarrow \sqrt{x} = \frac{1}{y}, x = \frac{1}{y^2}, \dots)$

$$\frac{1}{x^3} = \left(\frac{1}{y}\right)^3 = (y^2)^3 \dots$$

$$? D_f : \quad 5 > 3x^2 - \Leftrightarrow 5 > |x| \Leftrightarrow |x| < 5 \Leftrightarrow \textcircled{1}$$

$$f(x) = \frac{\log(|x|+4)}{\arccos(x^2-2)}$$

$[5, 5] = \text{Q}$

$$\left. \begin{array}{l} \text{1. } |x|+4 > 0 \Leftrightarrow |x| > -4 \\ \text{2. } \arccos(x^2-2) \neq 0 \\ \text{3. } -1 \leq x^2-2 \leq 1 \end{array} \right\} \quad \begin{array}{l} \text{1. } |x| > -4 \Leftrightarrow \forall x \in \mathbb{R} \\ \text{2. } x^2-2 \neq 1 \Leftrightarrow x^2 \neq 3 \Leftrightarrow x \neq \pm\sqrt{3} \\ \text{3. } -1 \leq x^2-2 \leq 1 \Leftrightarrow 1 \geq x^2 \geq -1 \end{array} \quad \text{Q.E.D.} \quad \textcircled{2}$$

$$1) |x| > -4 \quad \forall x \in \mathbb{R}$$

$$2) x^2 - 2 \neq 1 \Leftrightarrow x^2 \neq 3 \Leftrightarrow x \neq \pm\sqrt{3}$$

$$3) -1 \leq x^2 \leq 3 \quad \left. \begin{array}{l} x^2 - 1 \geq 0 \Leftrightarrow x \leq -1, x \geq 1 \\ x^2 - 3 \leq 0 \Leftrightarrow -\sqrt{3} \leq x \leq \sqrt{3} \end{array} \right.$$

$$-\sqrt{3} \quad -1 \quad 1 \quad \sqrt{3}$$

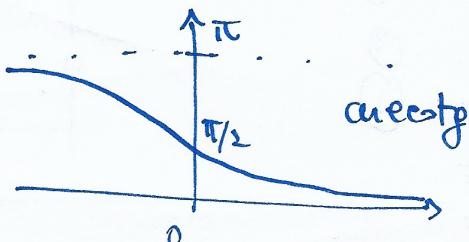


$$D_f = [-\sqrt{3}, -1] \cup [1, +\sqrt{3}]$$

$$f(x) = \frac{\log(3-|x|)}{\arctg(2\sqrt{x^2-x+1} + x)}$$

$$\left. \begin{array}{l} \log(3-|x|) \geq 0 \quad \textcircled{1} \\ 3-|x| > 0 \quad \textcircled{2} \\ \arctg(2\sqrt{x^2-x+1} + x) \neq 0 \quad \textcircled{3} \\ x^2 - x + 1 \geq 0 \quad \textcircled{4} \end{array} \right.$$

Si ricordi che :



$$\textcircled{1} \Leftrightarrow 3 - |x| \geq 1 \Leftrightarrow |x| \leq 2 \Leftrightarrow -2 \leq x \leq 2$$

$$\textcircled{2} \Leftrightarrow |x| < 3 \Leftrightarrow -3 < x < 3$$

$$\textcircled{3} \quad \forall x \in \mathbb{R} \quad (s-x) \text{ muss } s$$

$$D_f = [-2, 2]$$

$$\textcircled{4} \quad \Delta = -3 < 0 \Rightarrow \forall x \in \mathbb{R}$$

$$f(x) = \arctg \left(\frac{\sqrt{5^{2x}-3}}{2-\log x} \right)$$

$$\begin{cases} 5^{2x} - 3 \geq 0 & \textcircled{1} \\ 2 - \log x \neq 0 & \textcircled{2} \\ x > 0 & \textcircled{3} \end{cases} \quad \textcircled{1} \Leftrightarrow 5^{2x} \geq 3 \Leftrightarrow 2x \geq \log_5(3) \Leftrightarrow x \geq \frac{1}{2} \log_5(3) = \sqrt{\log_5(3)}$$

$$D_f = [\sqrt{\log_5 3}, +\infty] \setminus \{e^2\}$$

perché

$$0 \quad \sqrt{\log_5 3}$$

- \textcircled{1}
- \textcircled{2}
- \textcircled{3}

