COURSE OF STUDY **PHYSICS**
ACADEMIC YEAR **2023-24**
ACADEMIC SUBJECT **GEOMETRY**

General information	
Year of the course	First year
Academic calendar (starting and ending date)	First semester
Credits (CFU/ETCS):	9
SSD	MAT/03
Language	Italian
Mode of attendance	According to the study plan

Professor/ Lecturer	
Name and Surname	Antonio Lotta
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Department and address	Department of Mathematics, third floor, room n.2
Virtual room	Microsoft Teams
Office Hours (and modalities: e.g., by appointment, on line, etc.)	By appointment via e-mail

Work schedule			
Hours			
Total	Lectures	Hands-on (laboratory, workshops, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
225	56	30	139
CFU/ETCS			
9	7	2	

Learning Objectives	Acquisition of the fundamental notions of linear algebra and affine and euclidean geometry: matrix calculus and linear systems, vector spaces and linear applications, eigenvalues, eigenvectors and diagonalizability of endomorphisms, scalar products, affine spaces, Euclidean spaces.
Course prerequisites	Basic mathematical knowledge: polynomials, first and second degree equations and inequalities, fundamental theorems of euclidean geometry, elements of trigonometry, elements of analytic geometry in dimension 2.

Teaching strategy	Lectures and exercises. Exercise sheets will be provided.
Expected learning outcomes in terms of	
Knowledge and understanding on:	Acquiring fundamental concepts in linear algebra, dealing with vector spaces, linear maps and scalar products, and the basics of affine and euclidean geometry. Acquiring basic mathematical proof techniques.
Applying knowledge and understanding on:	The acquired theoretical knowledge is applied in solving problems in linear algebra and geometry, where students are particularly concerned with matrix calculus, systems of linear equations, bases and dimension of vector spaces, kernel and image of linear maps, diagonalization of endomorphisms or matrices, description

	of subspaces (linear, affine or euclidean) through parametric or Cartesian equations.
Soft skills	• *Making informed judgments and choices* Ability to analyse the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. • *Communicating knowledge and understanding* Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyse and solve problems. • *Capacities to continue learning* Acquiring suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested during the course.
Syllabus	
Content knowledge	**Elements of set theory.** Union and intersection of sets, complementary set, set of all subsets. Ordered pairs and ordered n-ples. Cartesian product of sets. Relationship between two sets. Functions. Direct image and inverse images of sets. Surjective, ingective and bijective mappings. Composition of maps. Bijective maps and their inverse. **Algebra basics.** Operations on a set. Groups and fields. Examples. The field of complex numbers. Algebraic representation of complex numbers. Conjugate and modulo of a complex number. Powers and roots of complex numbers. **Vector spaces and linear mappings**. Vector spaces over a field and their elementary properties. The vector space K^n. Direct product of two vector spaces. Linear mappings. Examples. Composition of linear maps. Isomorphisms. Vector subspaces. Kernel and image of a linear map. Monomorphisms and epimorphisms. Characterization of monomorphisms by means of the kernel. Sum of two subspaces, direct sum. Isomorphism between a direct sum of subspaces and their direct product. Classification of the linear spaces K^n spaces up to isomorphism. Vector subspace spanned by a finite set of vectors. Systems of generators. Finitely generated spaces. Linearly independent vectors. Bases of a vector space. Linear isomorphism K^n → V associated with a basis of a vector space V. Extraction of bases by the elimination algorithm. Notion of dimension of a vector space. Characterization of the dimension as the maximum number of independent vectors or minimum cardinality of a system of generators. The dimension of a subspace. The dimension of a direct product. Rank theorem for linear mappings. Existence and uniqueness theorem for linear maps. Grassmann formula. **Matrices and systems of linear equations.** The vector space of matrices with m rows and n columns over a field K. The transpose of a matrix. Square matrices, symmetric, skew-symmetric, diagonal matrices. Trace of a square matrix. Matrix product. Determinant of a square matrix: definition and properties. Invertible matrices and inverse matrix. The group GL(n, K) and its subgroups. Orthogonal matrices. Rank of a matrix: definition and properties. Matrix associated to a set of vectors with respect to a basis. Change of basis matrix. Matrices associated to a linear mappings. Systems of m linear equations in n variables. Cramer’s systems. Rouché-Capelli theorem. Homogeneous systems. General method for finding the solution set of a linear system.

	Eigenvectors. Eigenvectors, eigenvalues and eigenspaces of an endomorphism. Diagonalizable endomorphisms: definition and characterization. Similar matrices. Diagonalizable matrices. Characteristic polynomial. Algebraic and geometric multiplicity of an eigenvalue. Criterion of diagonalizability of endomorphisms. **Euclidean vector spaces.** Euclidean vector spaces. Scalar product. Standard scalar product on R^n. The norm of a vector. Convex angle between two non-zero vectors. Parallel vectors. Orthogonal vectors. Systems of orthogonal vectors. Orthonormal bases. Change of orthonormal basis matrix. Gram-Schmidt theorem. Orthogonal complement of a linear subspace. Unitary operators: definition, properties and examples. Unitary operators and orthogonal matrices. Symmetric operators and the Spectral Theorem. **Fundamental notions of affine and euclidean geometry**. Affine subspaces of R^n. Parallelism. Affine Grassmann formula. Affine frames. Equations of an affine subspace. Distance between points. Perpendicularity. Distance betrween points. Distance between affine subspaces. Basic facts about isometries of R^n and concerning the classification of plane isometries.
Texts and readings	E. Sernesi: Geometria 1, Bollati Boringhieri. E. Abbena, A.M. Fino, G.M. Gianella: Algebra lineare e geometria analitica, (Volumes 1 and 2), Aracne. G. Landi, A. Zampini: Linear Algebra and Analytic Geometry for Physical Sciences, Springer.
Notes, additional materials	
Repository	Lecture notes and exercise sheets available during the course.

Assessment	
Assessment methods	The exam consists of a written test and a subsequent oral test. Access to the oral exam if the written test is sufficient (minimum score 18/30). The written test consists of exercises based on techniques explained during the lessons. The oral exam focuses on the whole program, with particular emphasis on the definitions of the mathematical concepts covered by the course, the statements of the theorems, and the proofs of some key results. Each student who passes the written test in a round has the right to defer the oral exam to a subsequent round of their choice (by reservation), within the last round scheduled for the first year of the Study Plan.
Assessment criteria	*At the end of the course the following points will be evaluated:* • *Knowledge and understanding*: Knowledge of the fundamental concepts in linear algebra, affine and euclidean geometry, together with the capacity to state and prove related properties; capacity to show the acquired notions in specific examples. • *Applying knowledge and understanding*: Knowledge of how to use the acquired theoretical notions in solving exercises of linear algebra and geometry, including: matrix calculus, systems of linear equations, bases and dimension of vector spaces, kernel and image of linear maps, diagonalization of endomorphisms or matrices, parametric and cartesian equations of subspaces (linear, affine or Euclidean), with the description of related properties.

	• *Autonomy of judgement*: Capacity in evaluating the consistency of the logical arguments used in a proof. Problem solving skills, coherently with the acquired theoretical knowledge. • *Communication skills*: Capacity in the exposition of definitions, statements and proofs, and in presenting solutions of exercises in suitable mathematical language and formalism. • *Capacities to continue learning*: Capacity in consulting textbooks, in finding logical links and solving exercises.
Final exam and grading criteria	The final grade is out of thirty. The exam is passed when the grade is greater than or equal to 18/30.
Further information	
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