Dipartimento di Economia e Finanza

Course of Study: Statistics

Academic Year: 2023/2024

Academic Subject: Mathematical analysis and calculus of probability

General information	
Year of the course	Second Year
Academic calendar (starting and ending date)	First semester: from Sep. 11th 2023 to Dec. 15th 2023
Credits (CFU/ETCS):	10 CFU
SSD	MAT/05
Language	Italian
Mode of attendance	Attendence is not mandatory

Professor/ Lecturer	
Name and Surname	Giovanni Taglialatela
e-mail	giovanni.taglialatela@uniba.it
Telephone	
Department and address	Economics and Finance, Largo Abbazia Santa Scolastica - 70124 Bari
Virtual room	Microsoft Teams code x79mw1p
Office hours	By appointment, to be requested via email

Work schedule			
Hours			
Total	Lectures	Hands-on (laboratory, workshops, working groups, seminars, field trips)	Out-of-class study hours /Self-study hours
250	70		180
CFU/ETCS			
10			

Learning Objectives	The course aims to provide the basic tools of differential and integral calculus for functions of several variables and the main notions of the theory of probability, random variables and their indicators. These tools will enable the student to successfully attend the other classes and the subsequent professional statistician activity. The lessons are aimed at enhancing and refining the logical deductive skills and the critical sense of the student, to get him used to expressing himself with precision and language properties.
Course prerequisites	All the notions of the course of "Institutions of Mathematical Analysis" of the first year.

Teaching strategie	Theoretical lectures and exercises.
Expected learning outcomes in terms of	
Knowledge and understanding on:	The main notions and the most important results of differential and integral calculus for real functions of several variables as well as the basic theory of probability and random variables (discrete and continuous).
Applying knowledge and understanding on	The student will have to develop the ability to solve problems by applying the theorems, tools and methods learned in class.
Soft skills	The student must be able to critically evaluate different operational solutions in order to identify the most suitable for the objectives to be pursued.

Syllabus	
Content knowledge	**Combinatorcs.** Different ways to extract a sample of *k* elements from a set of *n* objects: ordered and unordered samples, with or without repetition. Arrangements, permutations, combinations. Factorial and binomial coefficients. Newton's binomial formula. Generalized binomial coefficients and multinomial coefficients. Combinatorial probability. **Notions on the cardinality of sets.** Countable and uncountable sets; countability of sets of natural, integer and rational numbers; uncountability of the set of real numbers. **Series.** Partial sums. Convergent, divergent and oscillating numerical series; sum of a series. Necessary condition for the convergence of a series. The geometric series theorem. Series with (definitively) positive terms. Harmonic series and generalized harmonic series. Criterion of comparison, asymptotic comparison, root and ratio. Absolutely convergent series. Series with terms of alternating sign; Leibniz's theorem. Power series: radius of convergence, derivation and term-by-term integration of a power series, Taylor series of some elementary functions. **Axiomatic theory of probability.** Set algebras and event logic. Equivalence between logical operations on events and operations on sets. Incompatible events and necessary events. Algebras and sigma-algebras of sets. Generated sigma-algebra, Borel sigma-algebra. Limit of a sequence of sets. Probability measure and related properties: probability of the complement, probability of the difference of two sets, inclusion-exclusion formula, monotonicity and continuity of the probability, Boole's and Bonferroni's inequalities. Almost certain events and almost impossible events. **Conditional probability.** Product rule. The law of total probability. Bayes theorem. Stochastic independence of two or more events. Positively or negatively correlated events. **Discrete random variables.** Distribution. Distribution function. Commonly used discrete distributions: uniform, Bernoulli, binomial, geometric, negative binomial, hypergeometric, Poisson. Mode, median, mean, moments, centered moments, variance, standard deviation of a discrete random variable. Independence between two or more random variables. Transformations of discrete random variables. **Improper integrals.** Examples and properties. (Improper) integration formulas by substitution and by parts. Comparison criterion and asymptotic comparison. The integral test for the convergence of a numerical series. **Continuous random variables.** Density. Distribution function. Commonly used densities: uniform, exponential, Cauchy, gamma, beta, normal, chi-square, student's *t*, Snedecor-Fisher's *F*. Median, mean, moments, centered moments, variance, standard deviation of a continuous random variable. Independence between two or more random variables. Transformations of continuous random variables. **Topology of $\mathbf{R}^n$.** Cartesian reference in the plane, in three-dimensional space,

	and in R^n. Operations between vectors: sum, product by a scalar and scalar product. Orthogonal vectors in R^n. Euclidean norm and distance in R^n. Open sphere, closed sphere and spherical surface. Internal, external, boundary and accumulation points for a subset of R^n. Open sets and closed sets of R^n: examples and properties. Bounded sets. **Scalar and vector values functions of several variables.** Image, graph and components of a function. Coordinate lines and contour lines for a function of two variables. Level surfaces for functions of *n* variables. Convergence and continuity for scalar or vector functions of *n* variables. Constant functions and projection functions are continuous functions. Limit theorems. Continuity of elementary functions of *n* variables. If *f* is a continuous function, the set of solutions of an inequality of the type $f(x)<c$ or $f(x)>c$, (resp. $f(x)\leq c$ or $f(x)\geq c$), is an open (resp. closed) set. A vector function is convergent (continuous) if, and only if, all of its components are convergent (continuous). Arc-connected sets in R^n. Bolzano' Theorem and Weierstrass's Theorem. **Differential calculus for functions of *n* variables.** Partial derivatives and directional derivatives, gradient and differential of a function of R^n in R. Differentiability. Necessary conditions for differentiability. Sufficient condition for differentiability. Tangent hyperplane and normal line to a level set of a differentiable function. Vector derivative of a function of a variable with vector values: its geometric meaning. Jacobian and differential matrix of a function of R^n in R^k. Differential of the composite function. Partial derivatives of higher order; Schwarz theorem on the invertibility of the order of derivation. Hessian matrix of a function of R^n in R. Taylor polynomial of the second order. Relative minimum and maximum points, saddle points: necessary conditions and sufficient conditions. (Strictly) convex or concave functions; examples and properties. Characterization of convex, concave, etc. functions. Convex functions and optimization. Constrained minimum and maximum points: Lagrange multipliers. **Integral calculus for functions of two variables.** Area of a rectangle and of a domain normal to an axis. Measurable sets according to Peano-Jordan and their measure. Riemann integrability for bounded functions on a measurable set of R2 and their integral. The integral as limit of Cauchy sums. Properties of the integral. Integrability of continuous functions in a normal domain with respect to an axis: reduction formulas. Integrability of continuous functions in a closed and measurable set. Integrability of generally continuous and bounded functions into a measurable whole. Change of variables in a double integral. Polar coordinates. Calculation of integrals by transformation into polar coordinates. Improper integrals in 2 or more variables. Integral of the Gaussian function. Notes on measurement and integration according to Lebesgue. **Multidimensional random variables.** Joint distribution and marginal distributions. Joint distribution function and marginal distribution functions. Conditional distributions. Transformations of discrete and continuous random variables. Characteristic values of multidimensional distributions: mixed moments, correlation coefficient, conditional expected values. Multivariate normal density. **Convergence of random variables.** Characteristic function, moment generating

	function and probability generating function. Sequences of random variables. Convergence in law (in distribution). Convergence of the hypergeometric distribution to the binomial. Rare event theorem. Central Limit Theorem. Markov and Chebychev inequality. Convergence in probability and the weak law of large numbers. Almost certain convergence and strong law of large numbers. Outline of other types of convergence.
Texts and readings	M. Brabanti, C.D. Pagani, S. Salsa: Matematica, Calcolo infinitesimale e algebra lineare, Zanichelli. G. Dall'Aglio, Calcolo delle probabilità, Zanichelli. D.M. Cifarelli, Introduzione al calcolo delle probabilità, McGraw-Hill.
Notes, additional materials	
Repository	

Assessment	
Assessment methods	Written and successive oral exam. The written exam consists in various exercises on different topics of the course. The oral exam consists in the discussion of the written test and the verification of knowledge on additional topics that are not covered by the written test: the definitions of the concepts and the statements of the theorems covered in the course are required. The proofs of the main results are also required. In order to be admitted to the oral exam it is necessary to pass the written exam.
Assessment criteria	The following aspects are equally valued • the knowledge *and understanding* of the notions of the course, • the ability to apply the notions learned to the resolution of exercises, • the a*utonomy of judgment,* • the ability to expose with precision and rigour, • the c*apacities to continue learning.*
Final exam and grading criteria	The final mark is a global evaluation of the written and oral exams.
Further information	