COURSE OF STUDY *Physics (LM-17)*

ACADEMIC YEAR *2023-2024*

ACADEMIC SUBJECT *Mathematical Methods of Physics*

General information	
Year of the course	1st
Academic calendar (starting and ending date)	1st semester: September – December 2023
Credits (CFU/ECTS):	6
SSD	FIS/02
Language	English
Mode of attendance	Recommended, not compulsory

Professor/ Lecturer	
Name and Surname	Prof. Paolo Facchi
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Department and address	Physics Department, via Amendola 173, Bari, office 182
Virtual room	
Office Hours (and modalities: e.g., by appointment, on line, etc.)	Students are invited to send an e-mail to arrange individual or group meetings

Work schedule			
Hours			
Total	Lectures	Hands-on (laboratory, workshops, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
150	24	45	81
CFU/ECTS			
6	3	3	

Learning Objectives	Acquisition of advanced mathematical methods for modern physics
Course prerequisites	Real and complex analysis, Fourier transform, Distribution theory, Quantum mechanics

Teaching strategy	Lectures and exercise sessions
Expected learning outcomes in terms of	
Knowledge and understanding	• Understanding the scientific method, the nature, and the methods of research in Physics • Knowledge of mathematical and probabilistic methods for physics • Knowledge of advanced mathematical tools commonly used in basic and applied research fields • Knowledge of advanced computational techniques • Knowledge of the advanced mathematical techniques commonly used in fundamental and applied research in physics. In particular, a knowledge of the mathematical structures of functional analysis and the theory of operators on Hilbert spaces, necessary for understanding advanced problems of modern physics.
Applying knowledge and understanding	• Ability to use analogy to apply known solutions to new problems (problem solving)

	• Ability to design and implement experimental or theoretical procedures to solve problems in academic and industrial research or to improve existing results • Ability to use analytical and numerical mathematical computation tools • Knowledge of general and advanced analytical and approximation techniques for understanding quantum phenomena and solving problems in quantum mechanics and quantum field theory.
Soft skills	• **Making informed judgments and choices** o Ability to work with increasing levels of autonomy, including taking responsibility in project planning and managing facilities. o Within the mathematical methods of physics, the student will be able to identify the best mathematical strategy for tackling specific physical problems. • **Communicating knowledge and understanding** o Competence in communication in Italian and English in advanced fields of Physics o The student will acquire mastery of the mathematical lexicon of modern physics and of quantum physics. • **Capacities to continue learning** o Acquisition of basic knowledge tools for continuous learning and knowledge updates o The student will develop an attitude to the continuous updating of mathematical techniques and skills in physics research.
Syllabus	
Content knowledge	**Metric spaces.** Definition. Examples. Open sets, closed sets, neighborhoods. Topological spaces. Continuous mappings. Dense sets, separable spaces. Convergent and Cauchy sequences. Completeness. Examples. Completion of a metric space. **Banach spaces.** Vector spaces. Normed spaces. Completeness and Banach spaces. Examples: finite dimensional spaces, sequence spaces, function spaces. Bounded linear operators. Continuity and boundedness. BLT theorem. Continuous linear functionals and dual spaces. Banach space of bounded linear operators. Examples. **Introduction to measure theory.** Lebesgue integral. Sigma algebras and Borel measures. Measurable functions. Dominated and monotone convergence. Fubini theorem. Examples: absolutely continuous measure, Dirac measure, Cantor measure. Lebesgue decomposition theorem. **Hilbert spaces.** Inner product. Euclidean and Hilbert spaces. Orthogonality, Pythagorean theorem. Bessel and Cauchy-Schwarz inequalities. Triangular inequality. Parallelogram law and polarization identity. Examples. Direct sum. Projection theorem. Riesz-Fréchet lemma. Orthonormal systems and Fourier coefficients. Orthonormal bases and Parseval's relation. Gram-Schmidt orthogonalization procedure. Isomorphism with l^2. Tensor product and product bases. **Linear operators on Hilbert spaces.** C∗-algebra of bounded operators. Normal, self-adjoint, unitary and projection operators. Baire's category theorem. Uniform boundedness principle. Uniform, strong and weak convergence. Some quantum mechanics. Unbounded operators. Adjoint. Symmetric and self-adjoint operators. Examples: multiplication and derivation operators. Essentially self-adjoint operators. Fundamental criteria of self-adjointness and essentially self-adjointness. Graph, closure

	and inverse of an operator. Self-adjoint extensions of positive operators. Example: kinetic energy in a segment. Self-adjointness of observables. **Spectrum and dynamics.** Resolvent operator, resolvent set and spectrum. Examples: position and momentum operators. First resolvent formula and analytic properties. Neumann series. Spectrum and Weyl sequences. Spectrum and eigenvalues of the inverse. Spectrum of self-adjoint, unitary and projection operators. Projection-valued measures and resolution of the identity. Integration on PVM of bounded functions. Expectation value of the resolvent. Spectral family of a self-adjoint operator and spectral theorem. Functional calculus. Spectral projections and spectral types. Quantum dynamics and unitary evolution groups. Energy conservation. Stone's theorem. Return and transition probability. Riemann-Lebesgue and Wiener Lemmas. Spectral types and return probability. Pure point spectrum and quasi periodic orbits. RAGE theorem.
Texts and readings	- M. Reed, B. Simon, Methods of Modern Mathematical Physics, Vol. 1, Academic Press, New York, 1980 - G. Teschl, Mathematical Methods in Quantum Mechanics, American Mathematical Society, Providence, 2009
Notes, additional materials	- Lecture notes
Repository	http://www.ba.infn.it/~facchi/Sito/Lectures.html

Assessment	
Assessment methods	Oral exam; written exercise
Assessment criteria	Capability to use techniques and solve problems introduced in the course. Adequate comprehension and global knowledge of concepts and arguments described throughout the course.
Final exam and grading criteria	Written exercise (50%). Oral exam (50%)
Further information	