Stampare su carta intestata del CdS

General information		
Academic subject	***CALCULUS I***	
Degree course	*Physics L30*	
Academic Year	*2021-2022*	
European Credit Transfer and Accumulation System (ECTS)		8
Language	*ITALIAN*	
Academic calendar (starting and ending date)		*From 2021-09-22 to 2021-12-17*
Attendance	*no*	

Professor/ Lecturer	
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Department and address	*Dipartimento Interateneo di Fisica*
Virtual headquarters	*Microsoft Teams*
Tutoring (time and day)	*Single, on demand on Microsoft Teams or more student in classroom after the lessons*

Syllabus	
Learning Objectives	*Acquire the basic notions of calculus: real numbers, the concept of limit, sequences and real functions, numerical series and integrals of one variable*
Course prerequisites	*Analytical Geometry, Logical and set-theory language, Operations between polynomials. Rational inequalities.*
Contents	*1. Real numbers* Hints of logic. Set theory: belonging, inclusion, union, intersection, complementary set, Cartesian product. What is an order relation. Natural numbers N, integers Z, rational numbers Q and their structures. No rational number has square 2. Finite, infinite and countable sets. Principle of induction. Bernoulli inequality. The real line, the intervals. Numerical sets: sup and inf, maximum and minimum. Field axioms of real numbers. The absolute value of a real number. Equivalent forms of the completeness axiom. Density of Q and its complementary in R. Archimedean property. *Examples of approximation of physical constants.* *2. Complex numbers* Complex numbers in algebraic form. Complex numbers in trigonometric and exponential form. Nth power of a complex number. Nth roots of a complex number. The fundamental theorem of algebra. *3. Elementary functions:* What is a function. Injective, surjective, bijective functions. Composition of functions, invertible functions and their inverse. Restriction and extension of a function. Direct and inverse image. The graph of a real function. Limited functions. Monotony, symmetries and periodicity of a function. Construction of some elementary functions, properties and graphs. Elementary operations on function graphs. Rational, irrational and transcendent inequalities. Theorem: Any strictly monotone real function is ingective. *Examples of functions describing periodic phenomena* *4. Numerical sequences:* What is a sequence. Regular sequences. Uniqueness of the limit. The expanded set R. Indeterminate forms. Operations on the limits of succession. Theorem: every convergent sequence is bounded. Theorems: sign permanence and conservation

	of inequalities for sequences. Comparison and ratio theorems for limit of sequences. Fundamental theorem on the limit of monotone sequences. Newton's binomial. Number of Napier. Remarkable limits of succession, scale of infinities. Subsequences and related theorem. Bolzano-Weierstrass theorem. Sequences defined by recurrence. *Examples of population dynamics models (the logistic map)* *5. Function limits:* Accumulation points and closed sets. Geometric examples (exhaustion) Function limits defined by sequences. Limit from right and left. Algebraic and comparison results for limits of functions, which are deduced from the same results for limits of sequences. Epsilon-delta rewriting of function limits. Operations on limits. Comparison theorems. Limits of elementary functions. Theorem: every convergent function is locally bounded. Notable limits. Infinites and infinitesimals and their properties. Principle of elimination of negligible terms. Horizontal and oblique asymptotes of a function. *Examples from population dynamics models (the exponential trend)* *6. Continuous functions:* Continuous functions and their elementary properties. The permanence of the sign theorem. When a function is not continuous at one point? Jump points, oscillations, continuity extension, vertical asymptotes. Continuous functions on intervals. The elementary functions are continuous in their domain. Zeros theorem. Weierstrass theorem. Intermediate value theorems. Existence of inverse of elementary functions. Link between monotony, continuity and invertibility. Continuity of the inverse function on intervals. Uniform continuity. Lipschtzian functions. Cantor's theorem. *Example of a continuous physical model of harmonic motion.* *7. Differential calculus:* Derivative of a function of a real variable. Continuity of differentiable functions. Theorem on the derivative of operations between functions (sum, product, quotient, composition) Derivative of the inverse function. Derivability of elementary functions. Angular points, cusp points. Local maximum and minimum points, critical points. Fermat's theorem. Rolle, Cauchy, Lagrange theorems. Monotony criteria. Functions with null derivative. Function with bounded derivative. De l'Hospital's theorem. Convexity for differentiable functions. Convex functions on an interval. Link between second derivative and convexity. Regularity of convex functions. Inflection points. Sufficient conditions for the existence of relative maximums, minimums. Taylor's formula with the remainder of Peano. Taylor's formula with Lagrange's remainder. Taylor expansions for elementary functions. Applications of Taylor's formula to classify maxima, minima and inflections. Study of the graph of a function. *Examples of geometric nature (tangent line) and kinematics (speed, acceleration).* *8. Integral calculation:* Partition of an interval. Upper and lower integral sums. Integrability according to Riemann. Characterization of integrable functions. Elementary properties of the definite integral. Integrability theorem of continuous and monotone functions. Mean theorem. Integral functions. Primitive and indefinite integral. Fundamental theorem of integral calculus. Structure theorem of the set of primitives of a continuous function. Torricelli's theorem. Methods for calculating indefinite integrals for rational functions. Integration by parts. Integration by substitution. *Geometric (area) and physical (work) meaning of the integral. First differential equations: homogeneous of the first order.* *9. Numerical series:*

	Definition of series and sum of series. The telescopic series (Mengoli series). The geometric series. Application of series to the decimal representation of real numbers. The harmonic series. Necessary condition for the convergence of a series. The character of a series does not change by altering a finite number of terms. Series with non-negative terms, dichotomy theorem. Simple comparison criteria. Criterion of the asymptotic comparison. The generalized harmonic series. Criterion of infinitesimals. Root criterion, relationship criterion. Absolutely convergent series are convergent. Alternating series. Leibnitz criterion for alternating series. The harmonic series with alternating sign. Interlocking sum. *Example of application to the concept of error* *10. Generalized integrals* Generalized integrals: integration of a function on a ray or of an unlimited function on a limited interval. The integral criterion for numerical series. Application to the generalized harmonic series. The Gamma function of Euler. *Application example: the sinc function and signals*
Books and bibliography	Some chapter from P. Marcellini & C. Sbordone -Elementi di Analisi Matematica 1– Liguori Editore, Napoli. E. Acerbi, G. Buttazzo – Primo corso di Analisi Matematica – Pitagora M. Bramanti -Esercitazioni di Analisi Matematica- Esculapio A. Alvino, C. Carbone, G. Trombetti -Esercitazioni di matematica Vol 1/1 Vol 1/2 - Liguori Editore.
Additional materials	Lecture Notes by the professor: https://www.sandralucente.it/didattica/appunti-lezioni Slide on Microsoft Teams

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
200	*40*	*45*	*115*
ECTS			
8	*5*	*3*	

Teaching strategy	
	Lectures with slides that are made in the classroom so that the explanation and understanding are aligned. The lessons made in the classroom are distributed at the end of the lesson on the Microsoft Teams platform. Classroom exercises.
Expected learning outcomes	
Knowledge and understanding on:	○ Know how to follows a math lesson, ○ know how to take notes, ○ know how to consult and understand university texts of Calculus, ○ Understand the resolution of exercises presented by teachers or by exercise texts
Applying knowledge and understanding on:	○ Review of basic knowledge. ○ Comparison of the topics of the course with the topics of the Physics course
Soft skills	• *Making informed judgments and choices* ○ Comparison between various demonstrations.

	○ Treatment of incoming data and critical analysis of the results in solving numerical problems • *Communicating knowledge and understanding* ○ knowing how to define, state and prove mathematical results ○ knowing how to explain to others your own resolution of an exercise • *Capacities to continue learning* ○ Acquire a study method that allows you to consult mathematics texts and keep the results in mind ○ Knowing how to choose exercises from the texts ○ Study theory and exercises at the same time

Assessment and feedback	
Methods of assessment	*Two passing written tests or a final written test. Then oral exam.*
Evaluation criteria	• *Knowledge and understanding* ○ Knowing how to consult their lecture notes and compare them with texts, discuss doubts and ideas arising from them with the teacher and possibly with classmates. • *Applying knowledge and understanding* ○ knowing how to graph elementary functions, be familiar with equations and inequalities • *Autonomy of judgment* ○ knowing how to evaluate the coherence of a logical reasoning. ○ Knowing how to choose the right mathematical tools to solve a given problem • *Communicating knowledge and understanding* ○ Ability to write an exercise paper that discusses the single steps ○ Ability to use the mathematical language • *Communication skills* ○ Ability to communicate their knowledge during the oral exam • *Capacities to continue learning* ○ During the written test we verify that the student is familiar with the techniques for the study of functions, the resolution of integrals, the discussion on the existence of limits and on the convergence of the series ○ During the oral exam we verify that the student knows theorems, definitions, examples (therefore exercises) and counterexamples and knows how to correlate them
Criteria for assessment and attribution of the final mark	The written test is passed if the student shows familiarity with each of the four proposed exercises. The oral exam can be held for the entire session in which the written exam is passed. The oral exam is passed if the student proves a theorem at the request of the teacher, knows how to explain the definitions and provide reasons for the hypotheses of some theorems. If the student has completely omitted the study of one part of the program, regardless of learning of the remaining part, the exam is not passed. The final mark depends on the mistakes made in the written test and on the ability to present the theory.
Additional information	